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A Discussion of Aperture and Space Allocation in the Energy Doubler/Saver

T.L. Collins and D.A. Edwards

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I. INTRODUCTION

In this paper, we discuss several of the conventional - as opposed to superconducting - accelerator aspects of the Energy Doubler/Saver (ED/S). The purpose is to examine how the aperture, tune, and space allocation for components may be influenced thereby.

The extraction system will be considered first; the requirement of efficient slow extraction places the most immediate demands for aperture in the extraction plane. Second, the question of the choice of tune will lead us to the conclusion that non-linear resonance effects are the major obstacle to a substantial reduction in tune below that of the main ring. Third, we will outline a specific lattice for the ED/S, including space allocated to a variety of adjusting and compensating elements as well as those appropriate to extraction and injection.

The ED/S design effort has naturally concentrated on superconducting magnets and cryogenic systems.[1] For the near future, it is probable that it will continue to do so. In that context, of the various arguments that we make here, only those concerning the aperture and field quality are applicable to the ED/S program as it is today. However, in our opinion, it is not too early to a study of other questions as well. While it would be absurd at this time for us to claim that we are presenting the lattice for the ED/S, we do urge that any ED/S permit the eventual accommodation of the functional features described below.

We take as our starting point the ED/S as described in Reference 1, including the two changes in location of dipole magnets that were proposed[2] to facilitate injection and extraction. As a result of the shift in bend centers, the ED/S will not be situated at the apex of the main ring enclosure, rather, the orbit will wobble back and forth with respect to the tunnel center. The excursions from the tunnel center line are listed in Table 1. The point of closest approach to the tunnel wall occurs at Station 2*, where the transition from the larger straight section enclosures takes place. After allowance for variation in floor elevation and local radius of the concrete hoops, an ED/S orbit 48" above that of the main ring is feasible provided the radial thickness of the ED/S magnet in the direction of the tunnel wall is not greater than 7.86". We believe this to be consistent with current ED/S designs.

The injected beam will be characterized by a gaussian distribution in both transverse planes. At 200 GeV, we take the rms beam height, σ_y, to be 0.04" at the maximum of β_y in the normal cell.[3] It will be assumed that at design intensity, the rms beam width will be twice this value - i.e. σ_x = 0.08" at the maximum of β_x in the normal cell. Unless otherwise indicated, the betatron oscillation tune is 19.4 in both planes.

*"Old" station numbers are used throughout this paper. Odd numbers correspond to positions of radially focussing quadrupoles. Service building entrances are at stations 5, 14, 22 and 30.

II. EXTRACTION

The ED/S Design Report indicates a preference for third-integral extraction; we base the discussion below on such a system. It is most unlikely that the aperture requirements and the needs for auxiliary components differ significantly from the half-integral case, however, an analysis of the latter as applied to the ED/S would be desirable.

A. Discussion of the Extraction Process

We begin with some qualitative remarks about the process. The resonance at ν_x = 19+1/3 is excited by a 58^{th} (3 x 19+1/3) harmonic in the distribution of sextupole fields around the ring. To initiate extraction, the horizontal tune is lowered from its nominal setting of 19.4 to a value close to 19+1/3; for example, ν_x = 19.34. The vertical tune is maintained at 19.4. The horizontal phase space then consists of two regions, as illustrated in either sketch in Figure 2(a). Particles of sufficiently small betatron oscillation amplitude will continue to undergo stable oscillation. However, any particle whose initial amplitude lies outside the triangular separatrix will describe an unstable oscillation that will eventually grow with time. As the tune is gradually shifted toward 19+1/3, particles migrate from the shrinking stable area into the unstable region and stream outward along the trajectories shown.

The arrows in the figure represent the direction of flow of the phase points. Suppose that the figures represent horizontal phase space at the location of the extraction septum. The 58th harmonic of the sextupole fields can have both a cosine amplitude, A, and a sine amplitude B. The sketches are drawn for pure A or pure B; mixtures of A and B amplitudes lead to triangles having orientations between the extremes shown. For highest efficiency, one desires the situation represented in Figure 2(b). Here a superposition of A and B terms is selected for which those particles having oscillation amplitude corresponding to the distance of the septum from the central orbit are directed parallel to the septum.

Figure 2(c) shows a later stage in slow extraction, with ν_x yet closer to 19+1/3. The triangle has been rotated, by changing the relative magnitudes of the A and B terms, to preserve the parallelism of the outgoing particles to the septum at the septum radius. In order for all particles to be extracted by this resonant process, ν_x must ultimately be set to exactly 19+1/3, at which time the stable area in phase space will have been reduced to zero.

The sextupole fields must be of sufficient strength to obtain an adequate step size (the increment in oscillation amplitude in three turns) at the septum radius to minimize the flux striking the septum. We take this desired step size to be equal to the septum radial aperture. The optimum step size is very slightly larger, since some particles can be allowed to strike the septum cathode in the interest of further reducing the flux at the anode; however, this difference is negligible for our purposes.

The various portions of Figure 2 show the lines bounding the stable area and the extensions of these lines as straight. Such is not necessarily the case. In fact, even in the absence of non-linearities other than the sextupoles, a slight curvature of the lines will develop due to effects which depend on the square of the sextupole moments. Of greater significance, however, is the presence of octupole fields - surely present in real magnets.

Figure 3 illustrates the influence of a zeroth harmonic octupole moment. In the first case, Figure 3(a), the octupole moment is sufficiently strong to cause substantial curvature of the lines, the curvature being positive or negative depending on the sign of the octupole moment. A somewhat larger octupole term - Figure 3(b) - will limit the growth of the otherwise unstable oscillation and lead to a degradation of the emittance of the extracted beam and/or a reduction of the extraction efficiency.

That a controllable admixture of octupole fields may be used to advantage is indicated in Figure 3(c). Here the appropriate amount of octupole is introduced so as to produce a near-parallel beam in the extraction channel.

For a quantitative treatment, we must first make assumptions concerning the available aperture. Currently, the ED/S group is considering two magnet apertures, both circular, having coil diameters of 3" and 2½". We will assume that allowances for vacuum chamber, straightness and out-of-round tolerances, and sagitta will reduce these figures by ½". Given excellent alignment, the r.m.s. orbit error will be assumed to be <1/8", with peaks <1/4".

Then, for the larger magnet, a 1" maximum amplitude betatron oscillation is possible. The extraction septum can be placed with its wires 0.6" from the axis; the anode will then be 1.0" off-axis. This is the arrangement shown in Figure 2(b).

For the 2½" magnet, the same considerations lead to 0.35" and 0.75" for the positions of the septum anode and cathode respectively, as shown in Figure 4.

It will be convenient to work in terms of the amplitude, a, and phase, ψ, of the betatron oscillation at the extraction septum. Since we want to take both sextupole and octupole nonlinearities into account, the magnetic field as a function of the transverse coordinate, x, and distance along the orbit, z, is .

$$B_y(z) = B_o(z) + a_2(z)x^2 + a_3(z)x^3 \tag{1}$$

$$a_2 = B''/2 \quad , \quad a_3 = B'''/6$$

The tune is close to the third-integral resonance, m, differing from it by a small quantity, δ, defined as

$$\delta \equiv \nu - m \quad ; \quad m = 19+1/3 \tag{2}$$

Then the equations expressing the rate of change per turn of amplitude and phase at the septum are, to the lowest order in the non-linear fields,

$$\begin{aligned} \frac{da}{dn} &= \frac{1}{4} a^2 \, (A \sin 3\psi + B \cos 3\psi) \\ \frac{d3\psi}{dn} &= 3\left[\frac{a}{4} (A \cos 3\psi - B \sin 3\psi) + \frac{3}{8} a^2 C + 2\pi\delta \right] \end{aligned} \tag{3}$$

where

$$A = \frac{\beta_o}{B\rho}\oint\left(\frac{\beta_x}{\beta_o}\right)^{3/2} a_2 \cos(3m\phi)dz$$

$$B = \frac{\beta_o}{B\rho}\oint\left(\frac{\beta_x}{\beta_o}\right)^{3/2} a_2 \sin(3m\phi)dz \qquad (4)$$

$$C = \frac{\beta_o}{B\rho}\oint\left(\frac{\beta_x}{\beta_o}\right)^{2} a_3 dz$$

In (4), β_o is the horizontal amplitude function at the septum and $\phi(z)$ is the "reduced" phase, $\int dz/(\nu\beta)$, which runs from 0 to 2π for one turn starting from the extraction septum. The harmonic coefficients A and B have the dimension of an inverse length - inches^{-1} in our units - while C will be in inches^{-2}.

The trajectory that a given particle will follow in phase space is found by eliminating n from the system (3), with the result

$$\frac{a^3}{3}\left(\frac{A}{4}\cos 3\psi - \frac{B}{4}\sin 3\psi\right) + \frac{a^4}{4}\left(\frac{3}{8}C\right) + \frac{a^2}{2}(2\pi\delta) = \text{constant} \qquad (5)$$

The separatrices of Figures 2 through 4 (plotted in coordinates x, $\beta_o x'$) are the trajectories for that particular value of the constant for which the curves pass through the fixed points at the vertices of the triangle.

The location of the fixed points (and the value of the constant) may be found by setting da/dn and d3ψ/dn equal to zero in (3). Then

$$\tan 3\psi_o = -B/A \qquad (6)$$

and the distance a_0 from the origin in the phase plane to the vertices is found by solving

$$\frac{3}{8} C a_0^2 - \left[\left(\frac{A}{4}\right)^2 + \left(\frac{B}{4}\right)^2 \right]^{1/2} a_0 + 2\pi\delta = 0 \tag{7}$$

We will refer to a_0 as the "corner amplitude."

An iterative numerical procedure was used to obtain the values of A, B, C and δ stated in Figures 2 through 4, which we need not describe here. An approximate treatment, which contains the main features, may be worth outlining. For this we will neglect the octupole term.

The step size is mainly dependent on B. If we set $\psi = 0$ in the first of the pair of equations labeled (3) and require that the amplitude grow from 0.6" to 1" in three turns, then integration yields

$$\frac{3}{4} B = \frac{1}{0.6} - 1 \Rightarrow B = \frac{8}{9} \text{ inch}^{-1}$$

That setting $\psi = 0$ as the amplitude grows through the range defined by the septum aperture is a rather good approximation for this purpose may be observed by comparing the values of B in Figures 2(b) and 2(c). The constancy of B suggests that the sextupoles providing this term will be turned on prior to extraction and not changed throughout the process.

Next, A is determined by requiring that the condition $\psi = 0$ at the septum wires be satisfied. Now we need to know the size of the triangle. At the outset of extraction, the phase space area within the triangle is the emittance of the entire beam; prior to the situation of Figure 2(b) the tune has gradually been lowered to approach the resonance and the phase space boundary of the beam has deformed from a circle to a triangle of the same area. If a_1 denotes the maximum amplitude of particles in the beam when far from resonance, then

$$a_0 = \left(\frac{4\pi}{3\sqrt{3}}\right)^{1/2} a_1 = 0.18 \text{ inch}$$

for a_1 = 0.12" corresponding to $2\sigma_x$ for the design intensity beam at 400 GeV. To obtain $\psi = 0$ at a = 0.6", the triangle must be rotated through an angle whose cosine is 0.5 a_0/0.6 = 0.15. Then from equation (6)

$$A = -B/\tan\,(3\,\cos^{-1} 0.15) = -\,0.43 \text{ inch}^{-1}$$

The excitation for the A sextupoles will be gradually reduced as extraction proceeds.

Finally, δ is determined by requiring that for given A and B, the tune be close enough to resonance to give the proper corner amplitude, a_0, already calculated above. From (7)

$$\delta = \frac{1}{2\pi}\left[\left(\frac{A}{4}\right)^2 + \left(\frac{B}{4}\right)^2\right]^{1/2} a_0 = 0.007$$

As already noted, δ is very slowly reduced from this initial value to zero if the entire beam is to be extracted by the resonant process.

It is interesting to compare the parameters given on Figures 2 and 3 with those of Figure 4. Though the aperture for the smaller magnet is only ½" less, the strengths of the non-linear elements have increased dramatically from those associated with the larger aperture.

In the above discussion, it was tacitly assumed that the small amplitude tune was the same for all particles. The natural chromaticity of the ED/S (due to the variation of quadrupole focal length with momentum) will be about -22, thus a momentum spread of 10^{-4} would imply a tune spread in the beam of .01.

The actual chromaticity may be considerably larger as a result of a non-zero average sextupole moment in the main ED/S magnets (see Chapter IV). For the 3" aperture case, extraction starts at δ = .007 and ends at δ = 0. The tune spread, however, insures that when δ = .007 for particles of one momentum, δ will already be 0 for particles of slightly higher momentum. Since at any instant, A, B, and C are set for a particular size and orientation of the triangular separatrix, the result will be a dilution of the phase space in the extracted beam. It is therefore attractive to provide chromaticity control during extraction, and we will return to this subject in Chapter IV.

B. System Components

(a) Electrostatic and Magnetic Septa

Space is provided for these elements in the ED/S Design Report, in the manner described in Reference 2.

(b) Orbit Adjusting Magnets

The present main accelerator extraction system has three pairs of dipoles to adjust the horizontal position and angle and the vertical position or the orbit at the extraction point. We will assume that the vertical pair can be dispensed with in the ED/S, since one can do the same job by quadrupole position adjustment. The horizontal position and angle magnets are in continual use, however, in the present system, and, in particular are used to bring the orbit close to the extraction septum for single turn extraction at the end of slow spill.

Possible locations for these magnets can be selected with the aid of Table 5, which gives the relative horizontal betatron phases around the ED/S ring calculated for ν_x = 19+1/3, ν_y = 19.4. Each pair of magnets should be close to an integral multiple of 180^o apart in phase, to localize the orbit distortion, and the two pairs should be reasonably orthogonal insofar as their effect at the septum location is concerned. For example, an angle adjustment could be performed by a pair located at F27 and A9 while a pair at F25 and A17 would be dominantly a position control.

The displacement required in the 3" aperture case is 0.6", thus at 1000 GeV, a magnet should produce 200 kG-inch. Each could be a 60" long 3.3 kG window frame of aperture large enough to surround the transfer line - that is, regions where the beam pipe is warm are not required.

(c) Spill Control Quadrupoles

Prior to the initiation of slow extraction, the tunes are adjusted to $\nu_x = 19.34$, $\nu_y = 19.4$ by the main system of tune adjustment (as yet unspecified - see Chapter IV). From then on the horizontal tune will be adjusted by a quadrupole system, under feedback control, to produce the desired spill rate and to compensate for undesired time structure in the main magnet excitation. Such a quadrupole system is installed in the present main accelerator.

The tune variation required is only ~0.01, which if accomplished by a single quadrupole would require a quadrupole having a strength about 3% that of one of the main quads. The elements of this system are therefore small.

(d) Non-linear Elements

These items are discussed in Chapter IV.

(e) Single-Turn Extraction Kicker

The same dipole move that permits a wire septum double the length of that in the main ring to be installed in the ED/S on either side of station F33 also provides space for a double strength kicker at E33. In the event that half-integral extraction is adopted for the ED/S, the fast kicker location would be at C33, as it is in the main ring.

III. CONSEQUENCES OF GREATLY REDUCED TUNE

Suppose that the betatron oscillation tune of the ED/S were reduced by 30 or 40 percent from that of the main ring. The quadrupoles will become shorter and a greater fraction of the orbit could be occupied by bending magnets. For any given maximum field in the dipoles, the ultimate energy reached by the ED/S will be higher. Are there any disadvantages to such a design?

That the answer is not to be found in linear orbit dynamics is well known - dramatic changes in beam size and probable orbit distortions do not follow from major tune changes. This is illustrated in Figures 5 and 6.

However, when one looks at non-linear phenomena, a different picture emerges, and it is here that the problems in considering a low tune are to be found. Two factors are at work. First, non-linear resonance widths increase rapidly as the phase advance per cell is reduced, and second, the non-linear terms in the fields of superconducting magnets will likely be larger than those in conventional magnets. These two topics are discussed in the following sections.

A. Field Variations from Construction Errors

We give estimates of the patterns of multipole fields that can be expected from errors in wire placement. An E-series (3") ED/S dipole magnet is used in the numerical examples. This is a two shell design with 31 turns in 72^{o} for the inner shell, and 21 turns in 37^{o} for the outer in both the upper and lower half planes. With a steel shield of 3½" radius, 3996 amperes gives 40 kilogauss.

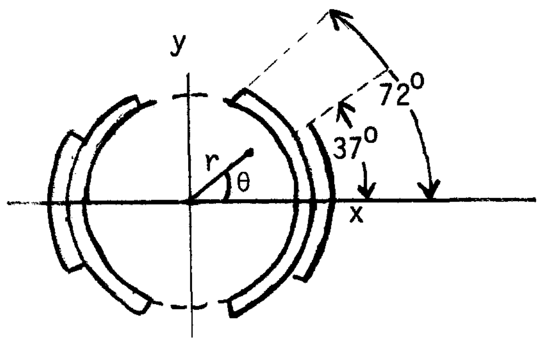
The multipole coefficients will be those appropriate to a scalar potential

$$\phi = B_o \sum_k \left\{ -\frac{\alpha_k}{k} r^k \sin k\theta + \frac{\beta_k}{k} r^k \cos k\theta \right\}$$

where B_o is the nominal dipole field and $\theta = 0$ at the midplane. At $y = 0$, for instance

$$B_y = B_o(\alpha_1 + \alpha_2 x + \alpha_3 x^2 + \ldots.)$$

$$B_x = -B_o(\beta_1 + \beta_2 x + \beta_3 x^2 + \ldots.)$$



The coefficients have the dimension of an inverse length to the (k-1) power. Here we will use the inch as the unit of length.

1. Azimuthal Errors. The table below presents three cases:
   (a) An rms position error $\langle\delta\rangle$* in placing each turn with respect to the previous turn.
   (b) The build-up in (a) is removed by squeezing to the correct final angle. The build-up error is re-distributed uniformly.

* To avoid a proliferation of superscripts, we write $\langle\delta\rangle$ instead of $\langle\delta^2\rangle^{1/2}$ in order to represent rms quantities.

(c) The coil is correct in cross-section but moved as a whole through a distance $\langle\delta\rangle$ from correct azimuthal position.

k	(a) $\langle\alpha_k\rangle$	(a) $\langle\beta_k\rangle$	(b) $\langle\alpha_k\rangle$	(b) $\langle\beta_k\rangle$	(c) $\langle\alpha_k\rangle$	(c) $\langle\beta_k\rangle$
1	1.14 $\langle\delta\rangle$	.75 $\langle\delta\rangle$	.25 $\langle\delta\rangle$	.36 $\langle\delta\rangle$	.14 $\langle\delta\rangle$	.21 $\langle\delta\rangle$
2	.45	.48	.36	.19	.18	.09
3	.27	.25	.26	.17	.11	.05
4	.15	.16	.14	.15	.04	.03
5	.09	.09	.09	.09	.02	---
6	.05	.05	.05	.05	.01	.01
7	.03	.03	.03	.03	.01	---
8	.02	.02	.02	.02	.01	---

2. Radial Errors. This table presents two cases:

(a) A random $\langle\delta\rangle$, independent from turn to turn

(b) A radial distortion of form

$$\Delta r = \frac{d}{k^2}\cos k\theta + \frac{e}{K^2}\sin k\theta$$

The k = 1 harmonic is not meaningful. The factor of $1/k^2$ represents the increasing strain energy of higher modes.

k	Radial $\langle\alpha_k\rangle$	Radial $\langle\beta_k\rangle$	Out-of-Round $\langle\alpha_k\rangle$	Out-of-Round $\langle\beta_k\rangle$
1	.031 $\langle\delta\rangle$	.020 $\langle\delta\rangle$	----	----
2	.037	.039	.055 d	.055 e
3	.035	.033	.028	.024
4	.025	.028	.009	.011
5	.020	.019	.008	.004
6	.014	.013	.003	.003
7	.009	.009	.002	.002
8	.006	.006	.001	.001

From the two tables above, one sees that squeezing the coil to the proper angle removes a big dipole and quadrupole error, that radial errors are an order of magnitude less important, and that the coefficients drop off rapidly above the octupole ($k = 4$) term. The last is a result of our using the inch as the unit of length and the E-series magnet - a two inch magnet, or an evaluation of multipole coefficients at half-aperture, would yield uniform values for all k.

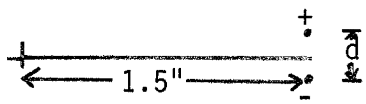
Azimuthal wire placement errors of only a few thousandths of an inch will yield multipole coefficients of 10^{-3} (in various powers of inverse inches) through the octupole term. Measurements on ED/S dipoles have found larger values of the quadrupole and sextupole terms, however, it seems reasonable to assume that conversion to the larger diameter E-series and gradual improvement in construction technique will reduce the low order multipoles to the 10^{-3} level.

Comparison with conventional magnets may be useful at this point. The main ring dipoles have a large systematic sextupole moment at injection field of approximately $6 \times 10^{-4}/\text{in}^2$. Due to variations in the coercivity of the steel, one might have a random sextupole term up to 50% of the systematic field. Thus, we anticipate random sextupole moments larger by a factor of three or more in the ED/S dipoles than in the main ring magnets - and they are present independent of excitation in contrast to the situation in the main ring magnets.

3. Fields from Leads. In current ED/S planning, the current leads running from magnet to magnet remain close to the beam pipe. We conclude this section with a tabulation of multipole coefficients arising from

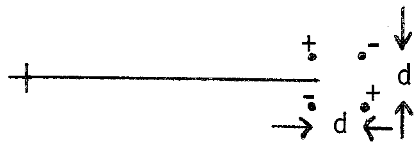
various lead configurations. The three cases in the table are sketched below; 4000 amperes is the current in each lead.

(a)



(b)

(c)

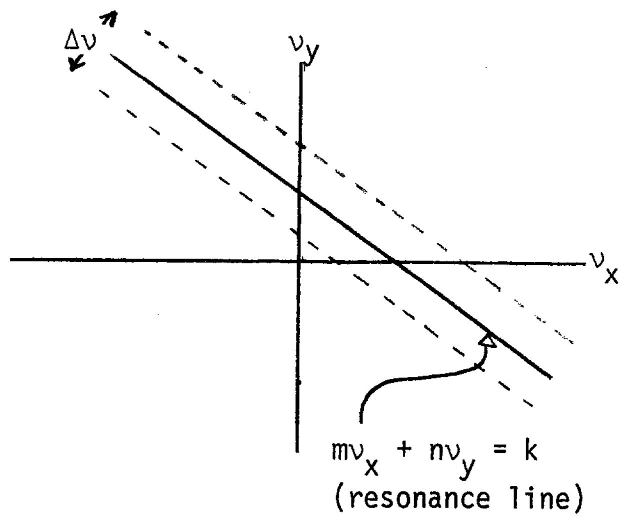


k	(a) $\langle\beta_k\rangle$	(b) $\langle\alpha_k\rangle$	(c) $\langle\beta_k\rangle$
1	.0029 d	.0027 d	.0032 d^2
2	.0035	.0032	.0056
3	.0031	.0028	.0065
4	.0025	.0022	.0064
5	.0019	.0016	.0056
6	.0014	.0011	.0046
7	.0010	.0008	.0036
8	.0006	.0005	.0027

One sees that fields due to the leads can be of some importance for a spacing ~1/4"(except for the symmetric four lead case).

B. Widths of Non-linear Resonances

There are a variety of ways of defining the width of a non-linear resonance. Here we will assume that a resonance is approached by adiabatic variation of the tune or of the resonance driving terms. The phase space area occupied by the beam will then deform as the separatrix closes in upon it. The total width, $\Delta\nu$, of the resonance will be the width of the band in the tune diagram such that for a small amplitude tune within the band 90% or less of the particles in a Gaussian beam will be within the separatrix. At the edge of the band, 10% of the particles of a Gaussian beam will be lost - for the real beam the fraction lost will be between 5% and 10% since the long tails of the Gaussian are not populated by particles.



The resonance width will be expressed as a product of the resonance driving term, the rms beam size, and a coefficient that includes the deformation of the beam phase space as the resonance is approached. The table below lists $\Delta\nu$ for resonances through 5th order (decapole fields) for a "round beam" ($\sigma_x = \sigma_y$) and for a beam with $\sigma_x = 2\sigma_y$ representing the eventual design intensity case.

Adiabatic Width of Resonances ($\Delta\nu_{TOT}$) for Total Range of $\Delta\nu$ with 90% or Less still inside Separatrix

Resonance	For $\sigma_x=\sigma_y=\sigma$	For $\sigma_x=2\sigma_y$		
$3\nu_x$	$.266\ \sigma A_K$	$.266\ \sigma_x A_K$	or	$.531\ \sigma_y A_K$
$2\nu_x + \nu_y$	$.495\ \sigma B_K$	$.408\ \sigma_x B_K$		$.816\ \sigma_y B_K$
$\nu_y + 2\nu_x$	$.495\ \sigma A_K$	$.375\ \sigma_x A_K$		$.751\ \sigma_y A_K$
$3\nu_y$	$.266\ \sigma B_K$	$.133\ \sigma_x B_K$		$.266\ \sigma_y B_K$
$4\nu_x$	$.327\ \sigma^2 A_K$	$.327\ \sigma_x^2 A_K$	or	$1.308\ \sigma_y^2 A_K$
$3\nu_x + \nu_y$	$.566\ \sigma^2 B_K$	$.406\ \sigma_x^2 B_K$		$1.624\ \sigma_y^2 B_K$
$2\nu_x + 2\nu_y$	$1.026\ \sigma^2 A_K$	$.586\ \sigma_x^2 A_K$		$2.345\ \sigma_y^2 A_K$
$\nu_x + 3\nu_y$	$.566\ \sigma^2 B_K$	$.235\ \sigma_x^2 B_K$		$.941\ \sigma_y^2 B_K$
$4\nu_y$	$.327\ \sigma^2 A_K$	$.082\ \sigma_x^2 A_K$		$.327\ \sigma_y^2 A_K$
$5\nu_x$	$.376\ \sigma^3 A_K$	$.376\ \sigma_x^3 A_K$	or	$3.01\ \sigma_y^3 A_K$
$4\nu_x + \nu_y$	$1.030\ \sigma^3 B_K$	$.696\ \sigma_x^3 B_K$		$5.57\ \sigma_y^3 B_K$
$3\nu_x + 2\nu_y$	$1.703\ \sigma^3 A_K$	$.693\ \sigma_x^3 A_K$		$5.55\ \sigma_y^3 A_K$
$2\nu_x + 3\nu_y$	$1.703\ \sigma^3 B_K$	$.584\ \sigma_x^3 B_K$		$4.67\ \sigma_y^3 B_K$
$\nu_x + 4\nu_y$	$1.030\ \sigma^3 A_K$	$.224\ \sigma_x^3 A_K$		$1.79\ \sigma_y^3 A_K$
$5\nu_y$	$.376\ \sigma^3 B_K$	$.047\ \sigma_x^3 B_K$		$.376\ \sigma_y^3 B_K$

The driving terms (the A_K or B_K) are defined by

$$A_K = \frac{\beta_o}{(B\rho)} B_o \oint \left(\frac{\beta_x{}^m \beta_y{}^n}{\beta_o{}^{m+n}} \right)^{1/2} \alpha_{m+n} \cos\left[\frac{k}{m+n}(m\phi_x + n\phi_y)\right] dz \text{ (max)}$$

$$B_K = \frac{\beta_o}{(B\rho)} B_o \oint \left(\frac{\beta_x{}^m \beta_y{}^n}{\beta_o{}^{m+n}} \right)^{1/2} \beta_{m+n} \cos\left[\frac{k}{m+n}(m\phi_x + n\phi_y)\right] dz \text{ (max)}$$

where α_j, β_j are the multiple coefficients defined in Section A, β_o is a reference value of the amplitude function at which σ_x and σ_y are calculated, and the designation (max) indicates that the starting point of the integration around the ring is shifted to maximize A_K or B_K.

In the case of random field errors, uncorrelated from magnet to magnet, the driving terms may be approximately by

$$\langle A_K \rangle = \frac{\beta_o}{(B\rho)} B_o N^{\frac{1}{2}} \ell \langle \alpha_K \rangle \quad \overline{\frac{\beta_x{}^m \beta_y{}^n}{\beta_o{}^{m+n}}}^{\frac{1}{2}}$$

where N (= 774) is the number of magnets, each of length ℓ. The expression for $\langle B_K \rangle$ is identical except for the replacement of $\langle \alpha_K \rangle$ by $\langle \beta_K \rangle$. Resonance widths for random errors are shown in Figures 7, 8, and 9, as a function of the phase advance per normal cell. Beam sizes are those appropriate to 200 GeV. The vertical dotted lines indicate the present normal cell phase advance of the main ring.

Note the rapid growth of the non-linear resonance widths as one goes to lower tune, and that the two dimensional resonances grow more rapidly than the one dimensional cases. Whereas 70^o per cell gives the minimum value of β_{max} for a particular cell length, consideration of non-linear effects points one toward a higher phase advance - near 90^o.

Rapid though the changes of widths with tune may be, are the magnitudes significant? Consider the third-integral case - Figure 7 . At $\nu_x=\nu_y$ = 19.4, with $\langle\alpha_3\rangle = \langle\beta_3\rangle = 10^{-3}/\text{inch}^2$, three of the rms widths are in the neighborhood of 0.01; the largest of the three is the $2\nu_x + \nu_y$ resonance with width 0.012. A reduction of the injection energy to 100 GeV will increase the widths by a factor of $\sqrt{2}$, to 0.017 in the $2\nu_x + \nu_y$ case. It is reasonably unlikely that any of the widths will be larger than two standard deviations; if one of the resonances were to have a width of 0.03, chances are that it could be identified and, if necessary, corrected. (It should be noted that the corrections implied are larger than those in the main ring.)

At a tune lower by 40%, the two dimensional third integral resonances would be a factor of two wider. As shown in Figure 5 , the quadrupoles would be some 22 inches shorter - if this 22 inches is used to increase the lengths of bending magnets, the maximum energy for a given dipole field would be 2.3% higher.

We find the results of this analysis encouraging. At a tune in the neighborhood of that of the main ring, and with realistic non-linear field tolerances, non-linear resonances will present a tractable problem at an injection energy as low as 100 GeV.

IV. ADJUSTMENTS AND CORRECTIONS

In the process of "tuning" a machine one needs the capability of making adjustments of certain parameters, which usually do not have a well-defined ideal value. Examples of adjustable parameters are

(a) The two betatron tunes, ν_x and ν_y

(b) The variation of betatron tunes with momentum (chromaticity)

(c) The variation of betatron tunes with amplitude

In Chapter II, several extraction adjustments were outlined; they are considered further in this chapter, for they are a part of the overall space puzzle that will be the prime problem below.

Other parameters do have an ideal value, frequently zero, but may differ from the ideal value due to field errors, survey errors and so on. Examples are

(a) Orbit distortions in general

(b) fine correction of the orbit at the injection septum

(c) the four third integral resonances.

The main ring contains an impressive array of adjustment and correction magnets. With the exception of the dipoles for correction of the low energy orbit, they were all fabricated and installed after the nominal end of main ring construction. This was possible since space for such components had been provided in the main ring lattice - the "mini-straight sections" at each quadrupole - without a predetermination of their nature and number.

Two factors make an earlier analysis of the space needs for these components appropriate in the case of the ED/S. First, as noted elsewhere in this report, any space they occupy might otherwise be devoted to bending magnets. Second, the "sardine-can" character of the cryo-magnet system argues in favor of inclusion of correction and adjustment magnets at construction time. We do not conclude, from this last point, that the correcting and adjusting fields be incorporated in the main dipoles and quadrupoles, though we suggest below that a systematic sextupole term to compensate the natural chromaticity be provided in the dipole coil configuration. To require more of the main magnets, however, would increase the complexity of their construction and run counter to the demand that they be obtained at lowest possible cost.

In designing the pattern of correction and adjustment elements, the following points should be kept in mind:

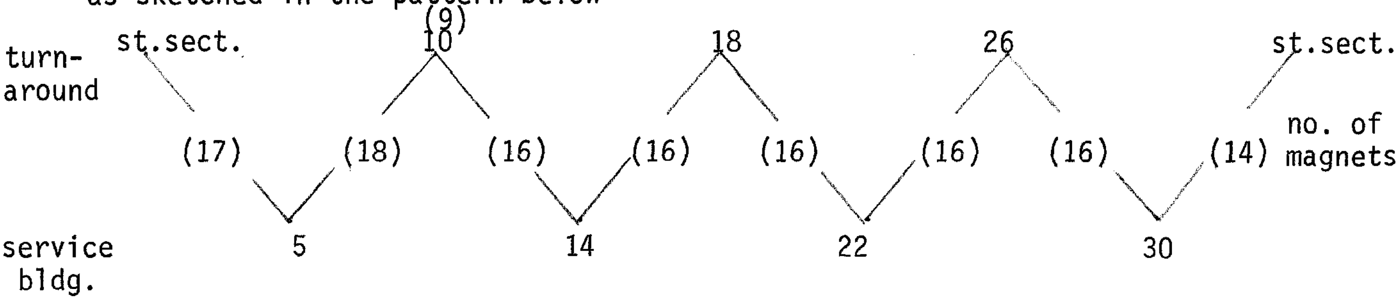
(a) The benefit of the change in a parameter is adjudged primarily by observation of one quantity: the total beam current. One must insure that the change in a parameter does not introduce a totally different error so that one cannot easily set the correction. A particular culprit in this regard is the mixing of orbit distortions with non-linear adjustments and corrections (as occurred in a most confusing fashion during the early operation of the main ring).

(b) The independence in (a) cannot be achieved solely as a problem of simultaneous equations to be solved by computer control. In general such an approach requires much stronger, largely cancelling elements for those corrections we intend to make and in addition requires new types of corrections for the new errors introduced by the adjustments.

(c) We must pay cryogenically for every lead and each amp entering cold boxes. There is a great premium on efficient production of necessary corrections.

A. Warm versus Cold Elements

Warm sections in the ring are cryogenically costly. In addition to the power lead thermal gradients there are the vacuum pipe thermal gradients. A certain number of warm places are necessary, particularly at the input and turn-around ends of the 48 magnet circuits. These warm short straight sections provide space for vacuum sectoring valves with major pumping facilities on each side. It would appear advisable to keep the strings of magnets in each cold section about the same length, as sketched in the pattern below



In one example of tune adjustment, we attempted to use the warm straight sections, in which case station 9 instead of 10 was used for a turn-around in order to get more F sections. However, it seems likely that the end boxes with their liquid He transport connections, major power leads and warm up section for the beam pipe will leave little surplus room after the vacuum equipment is installed.

Once one has decided that a cold pipe should exist at the correction elements, we have the choice of warm elements surrounding a (say) 6" cryostat pipe or superconducting elements.

Steel Elements 60" Long

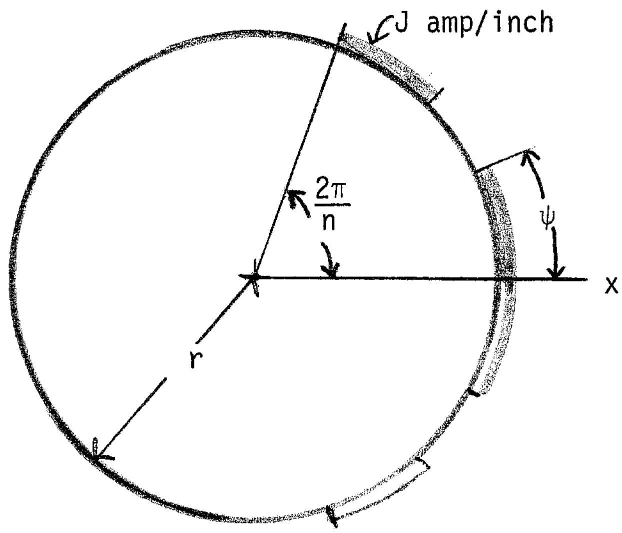
	Center of pole field	diam.	Maximum strength	Req'd approx.
steering dipole	18 KG	6"	18KG	3½ Kg
tuning quad (48)	12	6	4 Kg/m	2 Kg/in
(tuning quad)* (24)	12	main ring	6 Kg/m	6 Kg/in
sextupole	6	6	2/3 Kg/in^2	1½ Kg/in^2
octupole	3	6	1/9 Kg/in^3	2/3 Kg/in^3

*24 short main ring quads in warm straight sections.

The higher the multipole, the more difficult (or impossible) it becomes. Only the dipoles can fit easily at the ceiling location. Bus work is required.

Superconducting elements look more attractive. Take B_y to be of the form

$$B_y = a_1 + a_2 x + \dots a_k x^{k-1}$$



An "n-coil" magnet with cross section as sketched at the right will yield

$$a_k = \frac{\mu_o J}{2\pi r^{k-1}} \quad \frac{2n}{k} \sin k\psi$$

where

$$k = \frac{n}{2} \quad , \quad \frac{3n}{2} \quad , \quad \frac{5n}{2} \quad , \dots$$

Note that if ψ is selected so that the coil fills 2/3 of the available space, the second ($k = 3n/2$) term will vanish. There is a small penalty in field strength ($\sin k\psi = 0.866$), but some space in the coil is needed for winding anyway.

For a given current density, the strength of multipole magnets of this design falls by a factor of $1/r$ at each step from dipole to quadrupole to sextupole, etc. For $r = 1\frac{1}{2}$", this is a much slower fall-off than for the steel elements.

The table below lists a set of multipole magnets, consisting of single layer coils wound from 0.025" wire, allowing 0.026" per turn. The coil radius, $\bar{r}$, is $1\frac{1}{2}$".

	turns	width inches	ψ	AMPS	strength
dipole	60	1.56"	59.59^{o}	300	3.13 KG
quad	32	.832"	31.78^{o}	300	2.17 KG/inch
sextupole	21	.546"	20.86^{o}	300	1.43 KG/inch^2
(sextupole*	12	.480"	18.33^{o}	300	0.86 KG/inch^2)
octupole	16	.416"	15.89^{o}	300	0.96 KG/inch^3

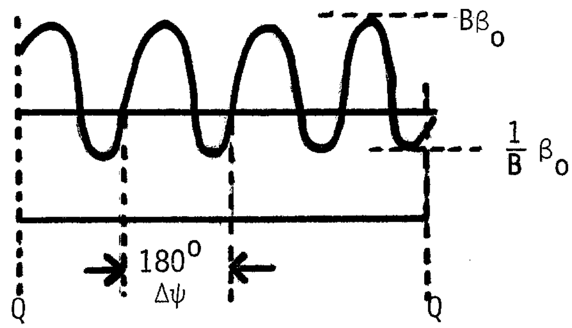
* .6 strength sextupole, .040"/turn, used in combinations (see below).

The dipole can be enhanced by a steel shield. At these currents, individual or small group power supplies are practical.

B. Betatron Tune Adjustment

A somewhat larger tuning range than that required for a resonant extraction and for quadrupole gradient length errors is a valuable diagnostic tool. We will design for a range of 19.0 to 19.75 along the $\nu_x = \nu_y$ "diagonal" and a tune split of up to 0.2.

A system with too few elements will cause excessive disturbance to the amplitude function. Between tuning quadrupoles, β develops a "beat" B. The



phase and amplitude of the beat changes at each tuning quad.

The beat is harmful not only for its effect on the beam size, but also in the confusion it creates. As one tunes to measure a resonance, the changing beat pattern selects a different mix of errors for the driving harmonic causing the resonance and also a different mix of correcting elements.

There are patterns of quad placement which produce large local beta beats without any tune shift (beta-bumps). The converse is not possible with a restricted number of quads; the best one can do is to achieve reasonable uniformity of the beat factor throughout the lattice. The beat improves rapidly as the number of tuning quads increases.

Three possible tuning arrangements were investigated:

(a) Separate circuits for the main quadrupoles, as in the main ring. A 2% change in current provides the tuning range with negligible beat. This would be the best choice; however, due to the many high current leads associated with distinct quad circuits, the ED/S design at present excludes this approach.

(b) Four quadupoles per superperiod at "warm" stations. In this case, F quads are placed at stations 5 and 9, D quads at stations 18 and 22. Short main ring quads installed at these locations could achieve the tuning range at full excitation (6000A). However, the beat is large, in excess of a factor of two, and the momentum dispersion is increased by a factor of 1.9 at certain places. Moreover, costly bus work would be required. This is not an attractive approach.

(c) Eight superconducting quadrupoles per superperiod.

The best pattern that we are able to devise for this case has F quads at stations 3, 13, 21 and 29, D quads at stations 2, 12, 20, 28. The beat is a maximum of 1.34 at $\nu_x = \nu_y = 19.0$. The dispersion function rises by 18% at most. Superconducting quadrupoles of the variety described in Section A could be used; with an effective length of 60", 2.09 KG/in gradient would be required.

We feel that case (c) represents an adequate tuning system, therefore provision for it is made in the overall space plan which concluded this chapter. In common with the other adjustment and correction elements, the excitation current is limited to 300A, thus the quadrupoles may be powered in local groups or two complete circuits may be installed using cables.

C. Non-linear Elements

Summary sheets for the several non-linear systems that should find accommodation in the lattice are included in this section. Before presenting these summaries, some remarks are in order.

With regard to chromaticity, the change of quadrupole focal length with momentum yields

$$\Delta\nu_x = \Delta\nu_y = - 22.08\ \Delta p/p \quad (\nu_x = \nu_y = 19.4)$$

That is, the "normal" chromaticity of the ED/S is - 22.08. Careful control of systematic sextupole field components, particularly in the magnet ends, is needed if the "normal" chromaticity is to be the largest contributor. Since a chromaticity near zero is usually preferred, that same care can be used to introduce intentional sextupole terms in the dipoles to cancel the normal chromaticity.

Two bending magnet types which differ in sign and magnitude of sextupole are implied. These will be designated as f and d dipoles; type f dipoles are those nearer an F quad, and type d are those nearer a D quad, except that all dipoles between station 33 and the doublet at station 34 are f-type and all dipoles between stations 1 and 2 are d-type. The tune shift from these sextupole terms is

$$\Delta\nu_x = \frac{\Delta p}{p}\,\frac{1}{2\pi\rho}\left[\alpha_f \int_f \beta_x \eta\, dz + \alpha_d \int_d \beta_x \eta\, dz\right]$$

where α_f, α_d are the sextupole components (inch^{-2}), and the expression for $\Delta\nu_y$ is obtained by replacing β_x with $-\beta_y$. Performing the integrals and solving for the α's required to eliminate the normal chromaticity yields:

(a) $\alpha_f = 2.65 \times 10^{-4}$ inch^{-2}; $\alpha_d = -3.48 \times 10^{-4}$ inch^{-2}

for a sextupole term uniform in a 240" magnet.

(b) $\int \alpha_f dz = .0323$ inch^{-1}; $\int \alpha_d dz = -0.0428$ inch^{-1}

for a sextupole term provided by the magnet ends only.

If these systematic values are achieved within ~20%, the chromaticity adjustment system given in the summary sheet will be adequate to compensate random field errors.

In principle, the three zeroth harmonics of octupole (involving β_x^2, $\beta_x\beta_y$, and β_y^2) should receive no contribution from systematic errors. An octupole implies an up-down or left-right asymmetry which is not present by design, however it could appear systematically in assembly by some tooling or procedure error. The octupole adjustment system specified here is predicated on a systematic octupole coefficient in the main dipoles of less than 2×10^{-5}/inch^3.

The patterns of non-linear elements have been selected so that each system will be as effective as possible in the performance of its intended function without introducing potentially harmful side effects. It is obvious that when adjusting sextupoles for chromaticity one does not wish to be making large changes in a third-integral resonance. But the sextupoles acting in conjunction with orbit distortions produce lower order effects as well. For example, a 20th harmonic in the chromaticity sextupole system can combine with a 19th harmonic orbit distortion to produce a 39th harmonic quadrupole term, thus opening a stopband at $\nu = 19\frac{1}{2}$. Unless some care is taken, the lower order effects can easily dominate.

An overall picture of the location of adjustment and correction elements is provided by Figure 10. At some positions, magnets combining two functions are assumed - two coil layers rather than one. The dipoles indicated in the figure are to provide angle and position control at the extraction septum. The two locations marked "inj." (injection) are for the injection septum and kicker.

0-HARMONIC SEXTUPOLE

$$A_x = \frac{\beta_o}{B\rho} \sum \frac{\beta_x}{\beta_o} \eta s \ell \qquad \Delta\nu_x = (A_x/2\pi)\ \Delta p/p$$

$$A_y = \frac{\beta_o}{B\rho} \sum \frac{\beta_x}{\beta_o} \eta s \ell \qquad \Delta\nu_y = -(A_y/2\pi)\ \Delta p/p$$

Location, in Sectors B and E

Stn. 3, 7, 11, 15, 19, 23, 27, 31 - 16 elements, strength S1 each

Stn. 4, 8, 12, 16, 20, 24, 28, 32 - 16 elements, strength S2 each

Relative Harmonic Amplitudes

0 - 16

18 - 1.4	56 - 6.3
20 - .68	58 - .08
22 - 2.0	60 - 3.3

For 1000 GeV, 60" long elements, S in KG/in^2

$$\left.\begin{aligned} A_x &= 392\ S1 + 81\ S2 \\ A_y &= 132\ S1 + 241\ S2 \end{aligned}\right\} \qquad \begin{aligned} S1 &= .00288\ A_x - .00097\ A_y \\ S2 &= -.00158\ A_x + .00468\ A_y \end{aligned}$$

Expectation Values

For $\langle\alpha_3\rangle = 10^{-3}$ random $\quad \left.\begin{aligned} \langle A_x\rangle &= 71 \\ \langle A_y\rangle &= 60 \end{aligned}\right\} \left\{\begin{aligned} \langle S1\rangle &= .21\ \text{KG/in}^2 \\ \langle S2\rangle &= .30\ \text{KG/in}^2 \end{aligned}\right.$

For $\alpha_3 = 5 \times 10^{-5}$ in all $\quad \left.\begin{aligned} A_x &= 88 \\ A_y &= 81 \end{aligned}\right\} \underbrace{\left\{\begin{aligned} S1 &= .18 \\ S2 &= .24 \end{aligned}\right.}_{\text{to correct to zero}}$

For standard sextupoles with 1.43 KG/in^2 at least .6 KG/in^2 will be available for adjustments or chromaticity range ± 20 (approximately "natural")

0-HARMONIC OCTUPOLE

$$A_x = \frac{\beta_o}{B\rho} \sum \left(\frac{\beta_x}{\beta_o}\right)^2 s\ell$$

$$A_{xy} = \frac{\beta_o}{B\rho} \sum \left(\frac{\beta_x \beta_y}{\beta_o^2}\right) s\ell$$

$$A_y = \frac{\beta_o}{B\rho} \sum \left(\frac{\beta_y}{\beta_o}\right)^2 s\ell$$

effect on tune

$$2\pi\Delta\nu_x = \frac{3}{8}\left[a^2 A_x - 2b^2 A_{xy}\right]$$

$$2\pi\Delta\nu_y = \frac{3}{8}\left[-2a^2 A_{xy} + b^2 A_y\right]$$

Location of Octupole Elements

Sectors	Stations	No. and Strength
A and D	19,23,27,31	8 at S1 KG/in^3
*All	1	6 at S2 KG/in^3
A and D	20,24,28,32	8 at S3 KG/in^3

Harmonic Amplitudes

0 - 8			*0 - 6
18 - 2.1	38 - .46	76 - .66	18 - 6
20 - 2.1	40 - .98	78 - .30	78 - 6 (bad)

For 1000 GeV, 60" long elements with S KG/in^2

$$\left.\begin{aligned} A_x &= 1.22\ S1 + .43\ S2 + .14\ S3 \\ A_{xy} &= .41\ S1 + .58\ S2 + .41\ S3 \\ A_y &= .14\ S1 + .78\ S2 + 1.22\ S3 \end{aligned}\right\} \begin{aligned} S1 &= 1.23\ A_x - 1.33\ A_{xy} + .31\ A_y \\ S2 &= -1.42\ A_x + 4.69\ A_{xy} - 1.42\ A_y \\ S3 &= .76\ A_x - 2.83\ A_{xy} + 1.68\ A_y \end{aligned}$$

Expectation Values

If $\langle\alpha_4\rangle = 10^{-3}$

$$\left.\begin{aligned} \langle A_x\rangle &= .37 \\ \langle A_{xy}\rangle &= .26 \\ \langle A_y\rangle &= .37 \end{aligned}\right\} \text{corr.} \begin{aligned} \langle S1\rangle &= .23\ KG/in^3 \\ \langle S2\rangle &= .36 \\ \langle S3\rangle &= .30 \end{aligned}$$

Systematic of 2×10^{-5} makes equal values.

1 KG/in^3 for S max, should be enough.

58 HARMONIC NORMAL SEXTUPOLE

$$\left.\begin{aligned} A_1 &= \frac{\beta_o}{(B\rho)} \sum \left(\frac{\beta_x}{\beta_o}\right)^{3/2} s\ell \;\; \cos 58\,\phi \\ A_2 &= \text{———————} \;\; \sin \text{——} \end{aligned}\right\} \text{drives } 3\nu_x$$

$$\left.\begin{aligned} A_3 &= \frac{\beta_o}{(B\rho)} \sum \frac{\beta_x^{1/2}\beta_y}{\beta_o^{3/2}} s\ell \;\; \cos 58\,\phi \\ A_4 &= \text{———————} \;\; \sin \text{——} \end{aligned}\right\} \text{drives } \nu_x + 2\nu_y$$

Location of Elements - each group has 8

sectors	stations	strengths	58th harmonic magnets phase
B and E	19,21,23,25	(.6,-1,1,-.6) S1	5.92 at 0^o
B and E	27,29,31,33	(.6,-1,1,-.6) S2	5.92 at 90^o
A and D	2, 4, 6, 8	(.6,-1,1,-.6) S3	5.92 at 180^o
*A and D	12,16,24,28	(.6,1,-1,-.6) S4	5.32 at 78^o

Harmonic Amplitudes

58 - 5.92		*58 - 5.32	
18 - .30	56 - 6.2	18 - .98	56 - 4.2
20 - .04	60 - 5.6	20 - .22	60 - 5.5
22 - .32		22 - .14	

For 1000 GeV, 60" long elements, S in KG/in^2

$$\left.\begin{aligned} A_1 &= .94\,S1 + 0 - .18\,S3 + .03\,S4 \\ A_2 &= 0 + .94\,S2 + 0 + .16\,S4 \\ A_3 &= .32\,S1 + 0 - .55\,S3 + .10\,S4 \\ A_4 &= 0 + .32\,S2 + 0 + .47\,S4 \end{aligned}\right\} \text{in}^{-1}$$

The sextupole element strength to make A's

$$S1 = 1.20\ A_1 + 0 - .40\ A_3 + 0$$

$$S2 = 0 + 1.20\ A_2\ \ 0 - .40\ A_4$$

$$S3 = .69\ A_1 - .15\ A_2 - 2.07\ A_3 + .44\ A_4$$

$$S4 = 0 - .81\ A_2 + 0 + 2.40\ A_4$$

Expectation Values

If $\langle\alpha_3\rangle = 10^{-3}$ (r.m.s.)

Then $\left.\begin{array}{r} \langle A_1\rangle = \langle A_2\rangle = .29/\text{in} \\ \langle A_3\rangle = \langle A_4\rangle = .25/\text{in} \end{array}\right\}$ $\langle S1\rangle = .36$ (probable sextupole strengths to correct the A's to zero)

$\langle S2\rangle = .36$

$\langle S3\rangle = .57$

$\langle S4\rangle = .64\ \text{KG/in}^2$

If the maximum S available is 1.43 KG/in^2 there is only a 4% chance that one S will not have enough strength.

<u>CAUTION</u>

If $\langle\alpha_3\rangle$ is allowed to rise to 2×10^{-3}, the probability is 50% that these elements are not sufficient.

58th HARMONIC CROSSED SEXTUPOLE

$$\left.\begin{aligned} B_1 &= \frac{\beta_o}{B\rho}\sum\left(\frac{\beta_y}{\beta_o}\right)^{3/2} c\ell \quad \cos\ 58\ \phi \\ B_2 &= \overline{\qquad\qquad\qquad} \quad \sin\ \overline{\quad} \end{aligned}\right\} \quad \text{drives } 3\nu_y$$

$$\left.\begin{aligned} B_3 &= \frac{\beta_o}{B\rho}\sum\frac{\beta_y^{\frac{1}{2}}\beta_x}{\beta_o^{3/2}} c\ell \quad \cos\ 58\ \phi \\ B_4 &= \overline{\qquad\qquad\qquad} \quad \sin\ \overline{\quad} \end{aligned}\right\} \quad \text{drives } 2\nu_x + \nu_y$$

Location of Elements - each group has 8

sectors	stations	strengths	58th harmonic magnets phase
A and D	19,21,23,25	(.6,-1,1,-.6) C1	5.92 at 0^o
A and D	27,29,31,33	(.6,-1,1,-.6) C2	5.92 at 90^o
F and C	2, 4, 6, 8	(.6,-1,1,-.6) C3	5.92 at 180^o
F and C	12,16,24,28	(.6,1,-1,-.6) C4	5.32 at 78^o

In all other respects, this system is the same as normal sextupole system.

THIRDS RESONANCE EXTRACTION ELEMENTS

Separate elements because they

(a) are strong.

(b) follow a different, complicated program.

(c) should not be required to go thru zero.

Septum Position	0^o Sextupole	90^o Sextupole	0-Harmonic Octupole
.6" max.	-.73/in	.88/in	$.42/in^2$
.35"	-3.69	1.87	2.98

Location - all elements in F and C - 8 in each group.

Type	Stations	Element Strength	Harmonic* Strength
Sextupole	7, 9,11,13	-.6,1,-1,.6	(58) + 1.35/in at 100^o
Sextupole	15,17,19,21	.6,-1,1,-.6	(58) - 1.35/in at 10^o
Octupole	7,11,15,19	1, 1, 1, 1	(0) $1.17/in^2$

*For standard correction elements, phases are measured at septum center.

V. CONCLUDING REMARKS

A lattice consistent with the foregoing discussion is summarized in the tables appended to this report. The "wobble" pattern - Table 1 - has been referenced earlier. As implied by the discussion in Chapter III, the nominal betatron tune in both transverse planes is 19.4. The normal cell of Table 2 is 1.05" shorter than that of the main ring, reflecting the lower harmonic number of 1112 instead of 1113. The "mini-straight sections" are one inch shorter, and, since the quadrupole in the ED/S is shorter than a main ring quad, a larger fraction of the cell is assigned to bending. We have matched the long straight sections for a tune of 19.4, hence the parameters of the long straight sections as listed in Table 3 are slightly different from those of the main ring (which is matched for ~20.25). A short summary of a number of quantities relating to the lattice is given in Table 4. The location of the adjustment and correction elements was worked out with the aid of a variety of phase tables - we include one of these here as Table 5, which is calculated for extraction tunes.

Time did not permit a serious consideration of the injection system. In Figure 10, space is set aside for the injection septum and kicker and, in Chapter IV, we have mentioned that control of the orbit position and angle will be needed at the septum. The dipoles performing these latter functions are small and positions can be found for them in the "mini-straight sections." What is lacking, however, is a study of the injection transport optics from the main ring to the ED/S and the implications of a vertical injection system for vertical aperture. In the brief time that we looked at the problem, it was not obvious to us how matching of amplitude and dispersion functions - both radial and vertical - could be accomplished between extraction and injection points. Low energy (~ 100 GeV) is clearly advantageous in this regard.

No provision was made for steering dipoles to correct closed orbit distortions in Chapter IV or in the layout of Figure 10. A complete system of steering dipoles is certainly unattractive even if they are to be effective only at injection energy. In conventional accelerators, they are included primarily because of remanent field effects - in the ED/S, with its range of choices for injection energy, no analogous phenomenon should occur. Also, the ED/S designers have proposed a magnet support system which will permit frequent adjustment of magnet position, possibly from the control room. Nevertheless, dipole rotations present a worrisome source of orbit distorting fields. A 1 milliradian rotation of a dipole magnet about its long axis will produce a radial field hence a vertical orbit distortion. For $\langle \Delta B/B \rangle = 10^{-3}$, Figure 6 shows that the rms orbit displacement at maximum beta in the normal cells will be 0.25". The chance that the peak-to-peak displacement will exceed a given multiple of the rms displacement is shown in Figure 11, where, for example, one sees that there is a 20% probability that the peak-to-peak displacement will be greater than 2.5 times the rms value.

If the dipole orientation were fixed "for all time" with the assembly of the machine, the orbit distortions from this source would be manageable. What worries us is the likelihood of orientation changes during warm-up and cool-down. A rotation of 1 milliradian corresponds to a twist of the coil by only 0.0015" at the coil radius. If rotations of even this magnitude are permitted to occur during temperature cycling, a formidable "orbit chasing" problem will arise.

In this paper, we have examined three of the conventional accelerator aspects of the ED/S - extraction, choice of tune, and the adjustment and corrections systems. Our conclusions may be summarized by three exhortations:

1. Provide sufficient aperture for slow extraction. (We feel that the E-series ED/S magnet promises to do so.)

2. Retain sufficiently strong focussing so that the non-linear fields likely in superconducting magnets do not become a severe performance limitation. (A betatron oscillation tune of 19.4 as in the main ring strikes us as a reasonable compromise.)

3. Reserve space in the lattice for adjustments and corrections. (The space need not be "warm"; a cold space comparable in length to the "mini-straight sections" of the main ring is appropriate.)

Finally, we point out that our discussion has been limited to systems situated within the ED/S ring; no mention has been made of how the protons may reach the machine, nor where they may go following their departure. The only treatment of these matters of which we are aware is contained in Reference 2, wherein the author, writing in the context of a summer study, has of necessity limited herself to a description of the central trajectory for injection and extraction and of the elements needed to establish that trajectory. We had hoped to carry out a more extensive investigation of the injection and extraction lines to include the focussing elements, but that has not been possible. Such an investigation is needed, and we recommend that it be authorized in the near future.

REFERENCES

[1] Energy Doubler/Saver Proposal, R. R. Wilson et al, August 1975.

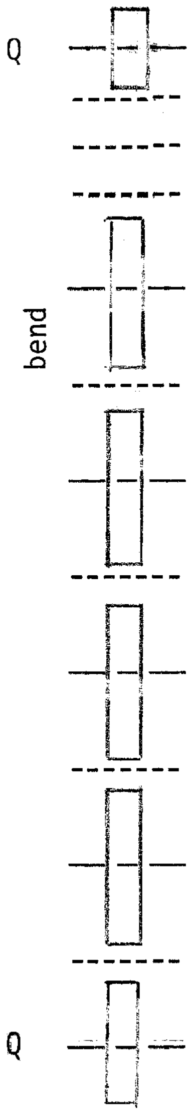
[2] H. T. Edwards, "Injection and Extraction for the Energy Doubler," 1973 Summer Study, Vol 2, Page 293.

[3] Beam sizes are based on measurements by L. Oleksiuk and collaborators on the fast extracted beam at 300 GeV.

	Med. St. Only	Long St. Only	Both		
0	3.76 inches	8.02 inches	10.41 inches	0	
1	3.93	14.47	17.0	1	added
*1	3.96	14.35	16.9	1	
2	4.24	13.43	16.2	2	
3	4.47	12.66	15.7	3	
4	4.69	11.88	15.1	4	
5	4.91	11.09	14.6	5	
6	5.13	10.29	14.0	6	
7	5.34	9.49	13.4	7	
8	5.38	9.32	13.3	8	added)
8	3.39	9.15	11.1	8	) med. st.
8	-0.64	8.97	6.96	8	removed)
8	-2.63	8.8	4.79	8	)
8	-2.6	8.68	4.7	8	
9	-2.39	7.86	4.09	9	
10	-2.17	7.03	3.49	10	
11	-1.95	6.2	2.88	11	
12	-1.73	5.37	2.26	12	
13	-1.5	4.53	1.65	13	
14	-1.27	3.69	1.04	14	
15	-1.04	2.85	0.43	15	
16	-0.81	2	-0.18	16	
17	-0.58	1.16	-0.79	17	
18	-0.34	0.31	-1.4	18	
19	-0.1	-0.53	-2.01	19	
20	0.14	-1.37	-2.61	20	
21	0.38	-2.21	-3.21	21	
22	0.62	-3.04	-3.8	22	
23	0.86	-3.88	-4.39	23	
24	1.1	-4.7	-4.98	24	
25	1.34	-5.53	-5.56	25	
26	1.59	-6.34	-6.13	26	
27	1.83	-7.15	-6.7	27	
28	2.07	-7.95	-7.26	28	
29	2.31	-8.75	-7.81	29	
30	2.55	-9.53	-8.35	30	
31	2.79	-10.3	-8.89	31	
32	3.03	-11.07	-9.42	32	
33	3.23	-11.71	-9.86	33	removed
33	3.27	-10.41	-8.52	33	
*34	3.55	0.28	2.46	34	
35	3.76	8.02	10.4	35	

* at quad nearest st. sect. center.

Table 1. "Wobble" in ED/S location with respect to tunnel center resulting from position shifts of dipole magnets proposed in Reference 2. First two columns show result of either shift alone. Displacements are measured at quadrupole midpoints, except for items within horizontal lines where displacements are given from the bend center of a dipole magnet.



	From F to D			From D to F		
ins	β ins.	ψ deg.	η* ins.	β ins.	ψ deg.	η* ins.
0	3918	0	156	1126	0	89
38	3835	.56	155	1152	1.92	90
	3706	1.09	152	1194	3.63	91
108	3580	1.64	150	1240	5.28	93
	3142	3.83	140	1430	10.79	99
364	2743	6.33	131	1659	15.56	105
	2382	9.20	123	1926	19.66	112
620	2060	12.51	115	2231	23.20	120
	1775	16.35	108	2575	26.27	128
876	1529	20.81	102	2957	28.92	136
	1321	25.97	95	3377	31.25	145
1132	1152	31.93	90	3835	33.28	155
1169.95	1126	33.84	89	3918	33.84	156

*Average values. η wobbles from cell to cell with betatron-like period.

Table 2. Parameters of the normal cell, for a quadrupole gradient of 20 KG/in and effective length of 63.742".

	Main Ring	Doubler		
Tune	20.3	19.4	19.4	19.4
Energy (GeV)	400	1000	1000	1000
Gradient	6.364	∞ *	20	16 KG/in
Quad	84	(f=1050.2)	63.74	80.06 in.
Short Quad	51.95	(.617314 f)	39.39	49.47 in.

½ quad *thin lens

	Main Ring	Doubler		
33 Short F				
	1154.98	1153.94	1153.94	1153.94
"Doublet" - 34: Short F				
	62.97	61.94	64.55	64.55
Short F				
	(A) 116.80	125.71	129.27	129.92
Short D				
	86.20	86.12	78.56	78.56
D				
	1042.675	1033.70	1035.09	1034.44
St.sect. center				
Total (33 Short F to St.sect. center)	2463.625			2461.41

Table 3. Long straight section parameters of the ED/S. Lattice is matched for tunes $\nu_x = \nu_y = 19.4$. Three columns at right list results for thin lens quadrupoles and for gradients of 20 and 16 KG/in. Present main ring parameters are shown for comparison.

Energy		1000 GeV		
Momentum		1000.938 GeV/c		
Bend Length		245" (eff)		
Field		43.554 KG		
Bℓ		10671 KG-in.		
Number of dipoles		774		
Half Cell Length		1169.95 in.		
Half Straight Section Length		2461.41 in.		
Quad Gradient		16	18	20 KG/in.
Length	0	80.06	70.97	63.74 in.
Gℓ	1251.6	1281.0	1277.5	1274.8 KG-in.
Short Quad Gℓ	772.6	791.4	789.4	787.8 KG-in.
Doub. Space A	125.71	129.92	129.56	129.27 in.
Tune	19.4	19.4	19.4	19.4
Phase/Cell	67.699	67.683	67.684	67.687^{o}
Phase/L.S.S.	114.67	114.91	114.90	114.85^{o}
Regular β_{max}	3938	3912	3915	3918 in.
β_{min}	1120	1128	1126	1126 in.

Table 4. A selection of lattice-related parameters for the ED/S. Lattice must be recalculated when final decision is made on quadrupole gradient. Note that for series operation of dipoles and quadrupoles, quadrupole effective length must be correct to $\lesssim$ 0.1 inch.

stn	F	A	B	C	D	E	F
0	- 28.8	51.2	131.2	-148.8	- 68.8	11.2	91.2
1	- 12.5	67.5	147.5	-132.5	- 52.5	27.5	107.5
3	67.7	147.7	-132.3	- 52.3	27.7	107.7	-172.3
5	135.4	-144.6	- 64.6	15.4	95.4	175.4	-104.6
7	-157.3	- 77.3	2.7	82.7	162.7	-117.3	- 37.3
9	- 89.8	- 9.8	70.2	150.2	-129.8	- 49.8	30.2
11	- 22.2	57.8	137.8	-142.2	- 62.2	17.8	97.8
13	45.1	125.1	-154.9	- 74.9	5.1	85.1	165.1
15	112.8	-167.3	- 87.3	- 7.3	72.7	152.7	127.3
17	-179.8	- 99.8	- 19.8	60.2	140.2	-139.8	- 59.8
19	-112.5	- 32.5	47.5	127.5	-152.5	- 72.5	7.5
21	- 44.8	35.2	115.2	-164.8	- 84.8	- 4.8	75.2
23	22.5	102.5	-177.5	- 97.5	- 17.5	62.5	142.5
25	90.0	170.0	-110.0	- 30.0	50.0	130.0	-150.0
27	157.6	-122.4	- 42.4	37.6	117.6	-162.4	- 82.4
29	-135.1	- 55.1	24.9	104.9	-175.1	- 95.1	- 15.1
31	- 67.4	12.6	92.6	172.6	-107.4	- 27.4	52.6
33	0	80	160	-120	- 40	40	120

Table 5. Relative phases of radial betatron oscillations at radially focusing quadrupoles for $\nu_x = 19+1/3$, $\nu_y = 19.4$ (situation at extraction). Phase at station F33, the extraction septum location, is taken as 0 in leftmost column.

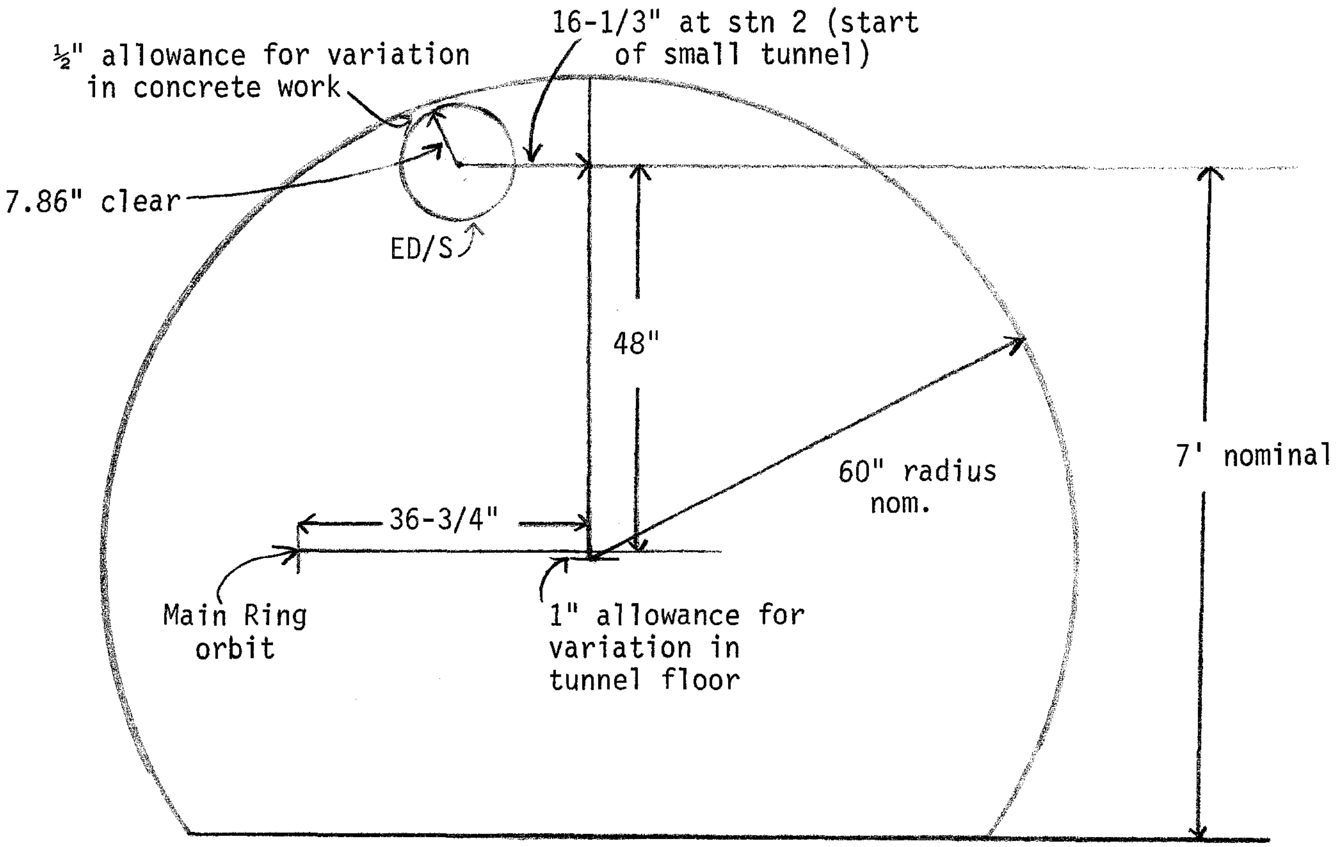


Fig. 1. Cross section through main accelerator enclosure at Station 2, where point of closest approach of ED/S to tunnel wall occurs. After 1" allowance for variation in elevation of floor and ½" for variation in concrete work of the wall, clearance remaining for ED/S is 7.86".

(a)

(b)

(c)

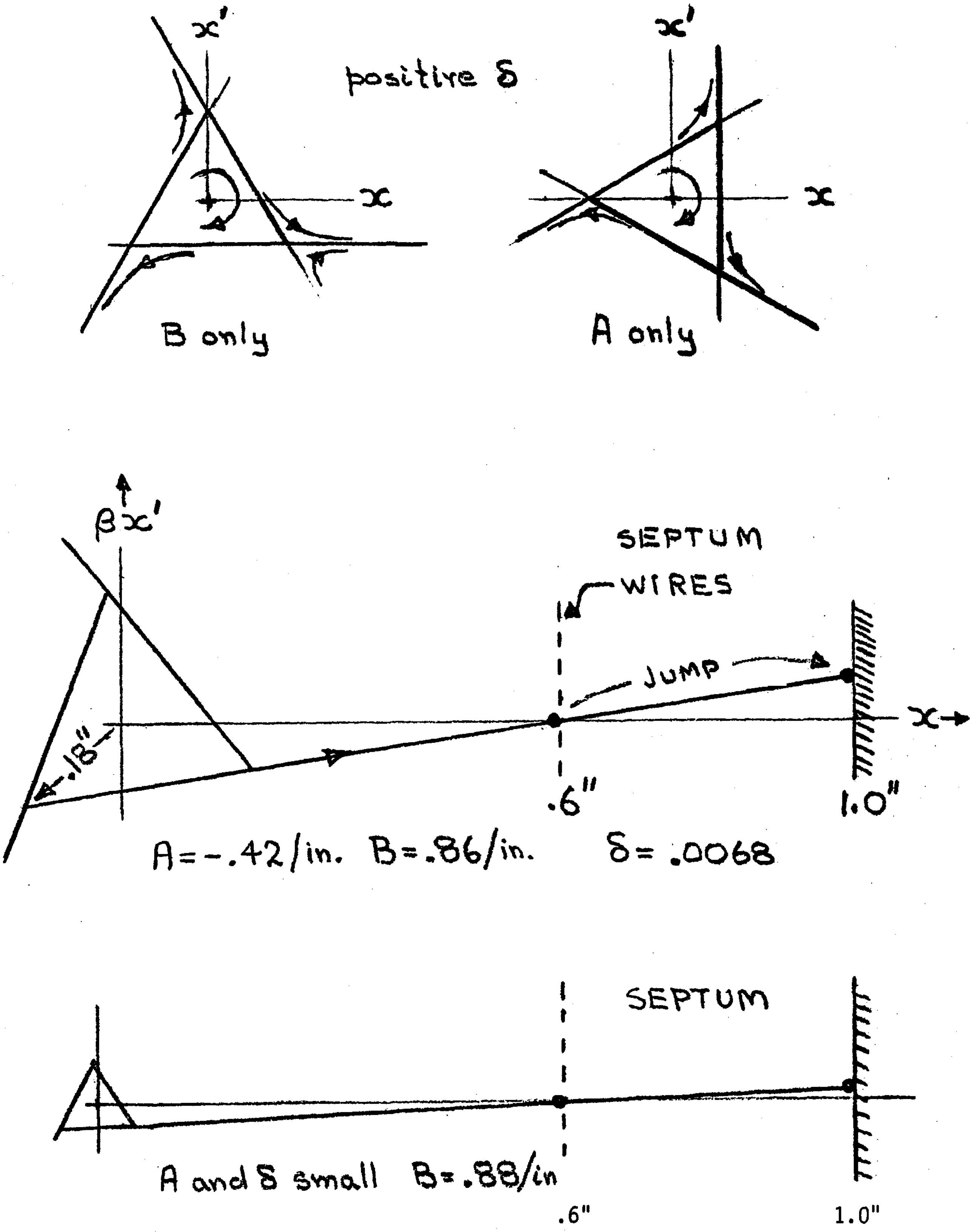


Fig. 2. (a) Separatrix for third-integral resonance - two azimuthal phases of driving term are shown. Arrows designate direction of flow of phase points.

(b) Phase space at beginning of slow extraction at 400 GeV in 3" coil bore magnet.

(c) At a later stage in slow extraction, the stable area in phase space is reduced toward zero by adjustment of the tune.

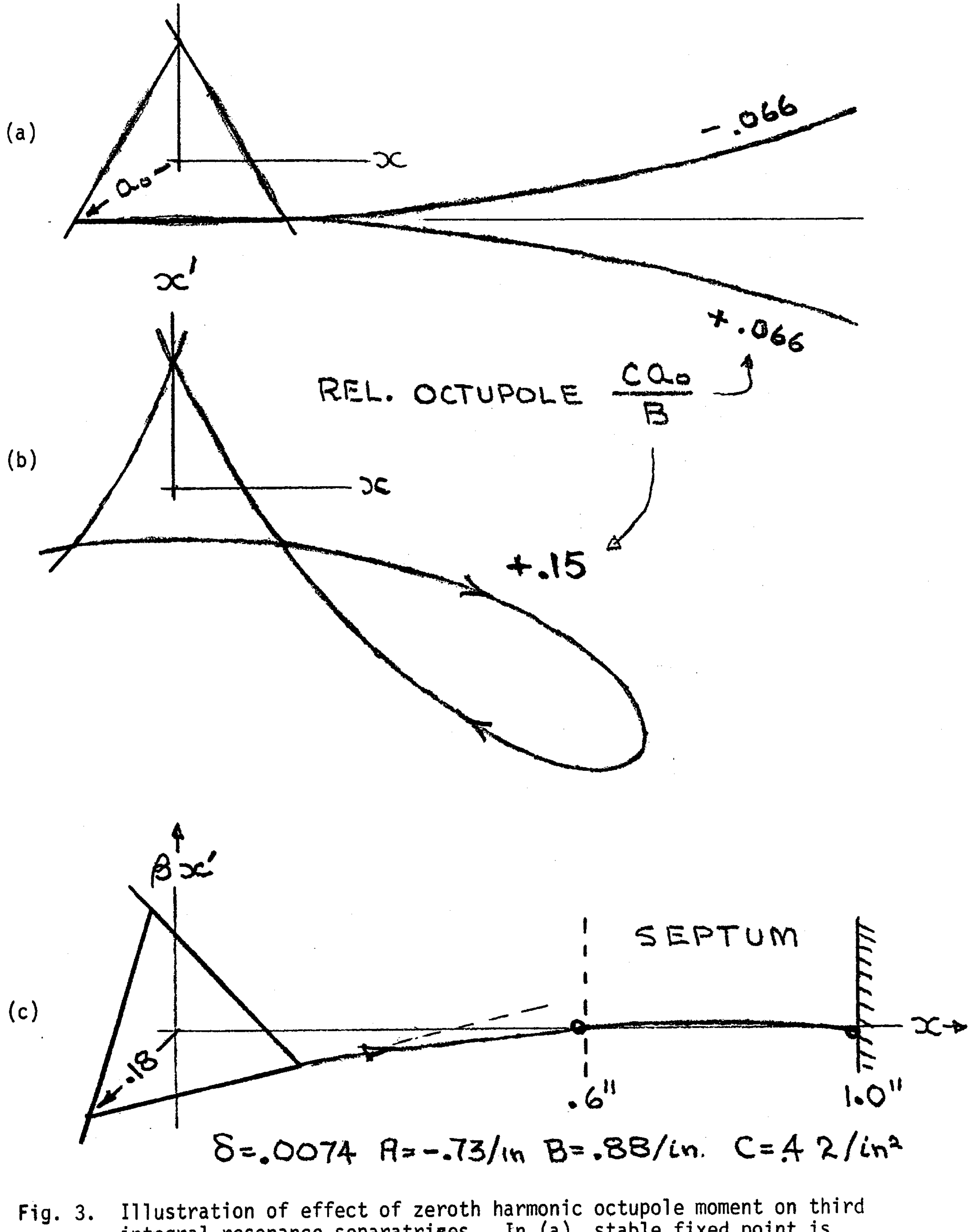


Fig. 3. Illustration of effect of zeroth harmonic octupole moment on third integral resonance separatrices. In (a), stable fixed point is outside of aperture; in (b), an overly large octupole moment "stabilizes" the resonance. (c), the octupole moment can be used to advantage to produce a nearly parallel beam at the extraction point.

(a)

(b)

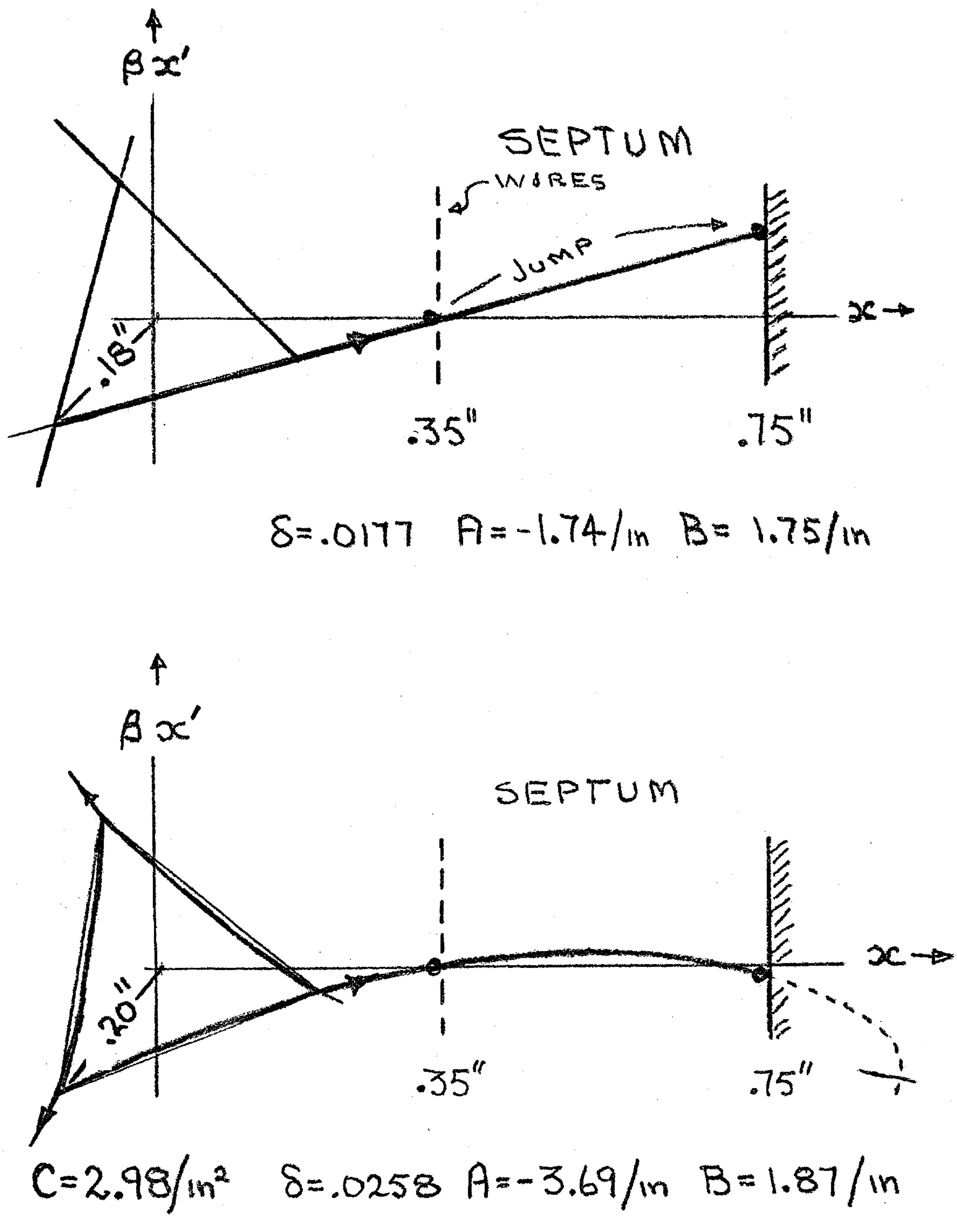


Fig. 4. Phase space diagrams for third-integral resonant extraction in 2.5" coil bore magnet, without and with zeroth harmonic octupole adjustment.

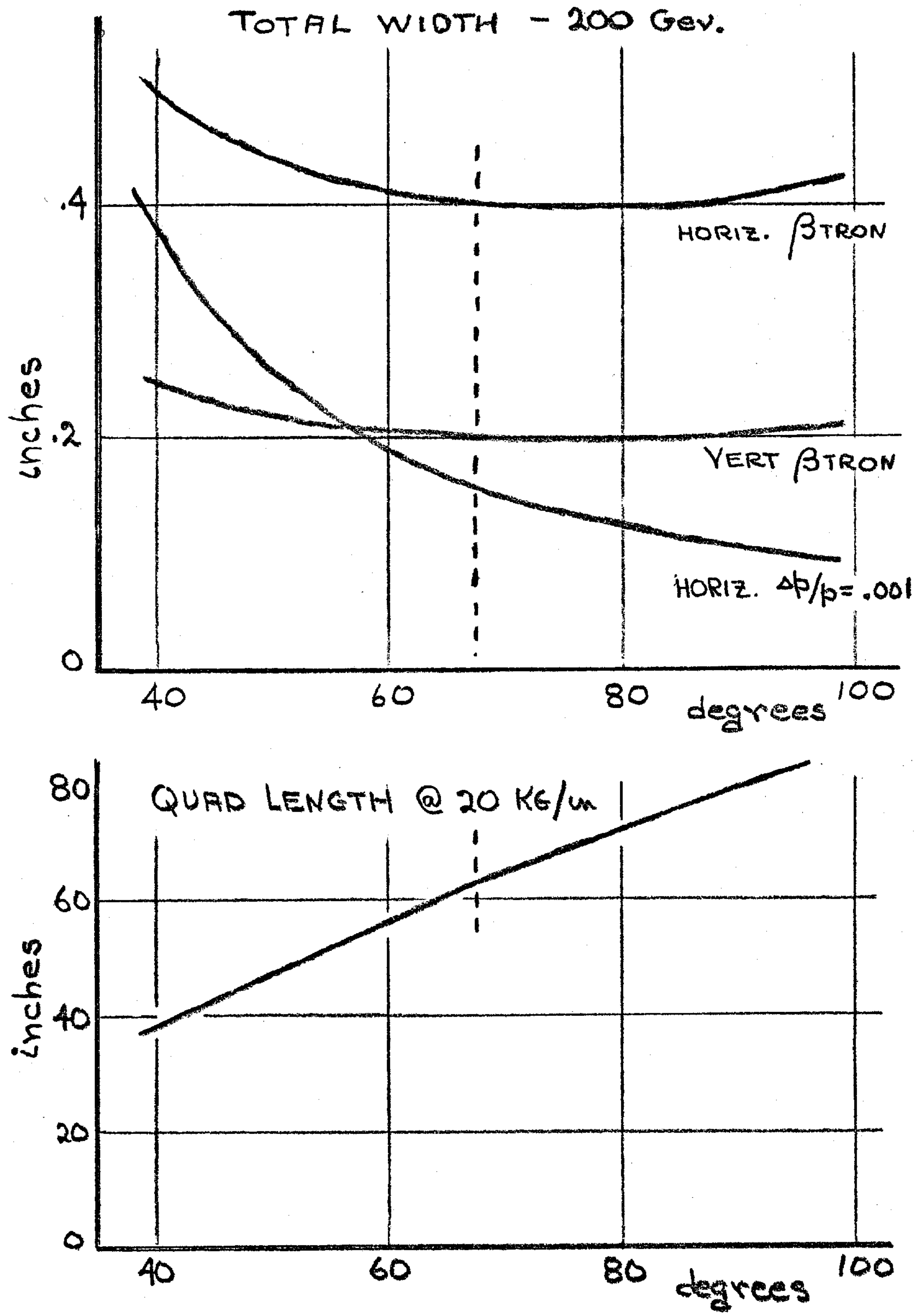


Fig. 5. The upper portion of the figure illustrates the variation of beam size with the choice of phase advance per normal cell. The vertical dotted line corresponds to a tune of 19.4. The length of a quadrupole of the normal cell is plotted versus phase advance in the lower portion. A gradient of 20 KG/inch has been assumed.

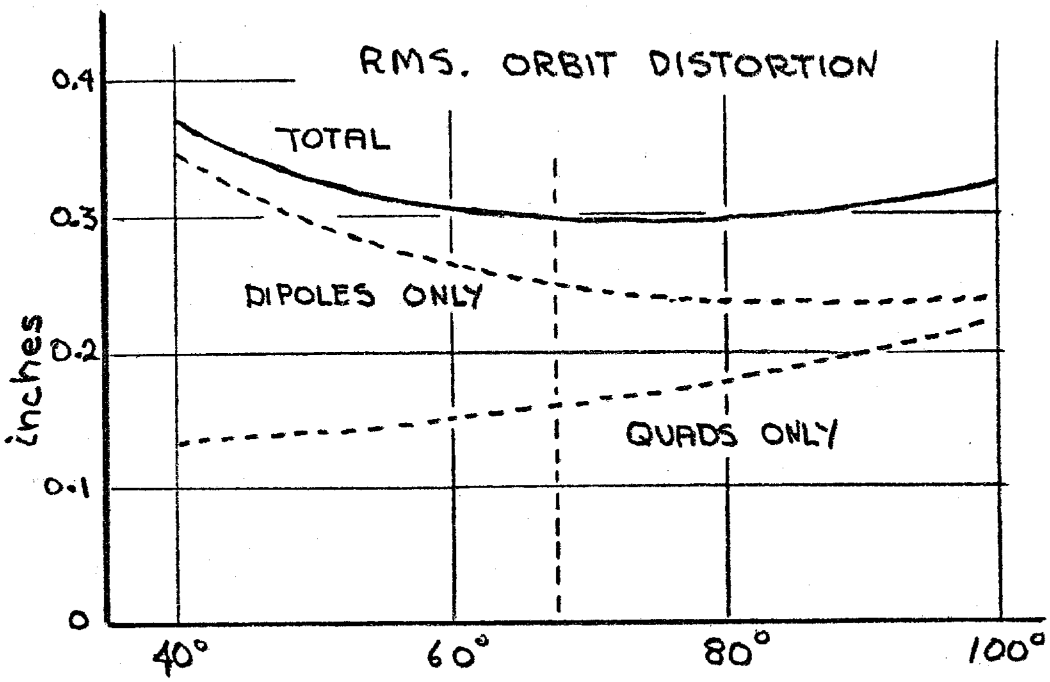


Fig. 6. Variation of rms orbit distortion with phase advance per cell. In the dipoles an rms fractional field error of 10^{-3} is assumed; for the quadrupoles, an rms placement error of 0.010 inches has been used.

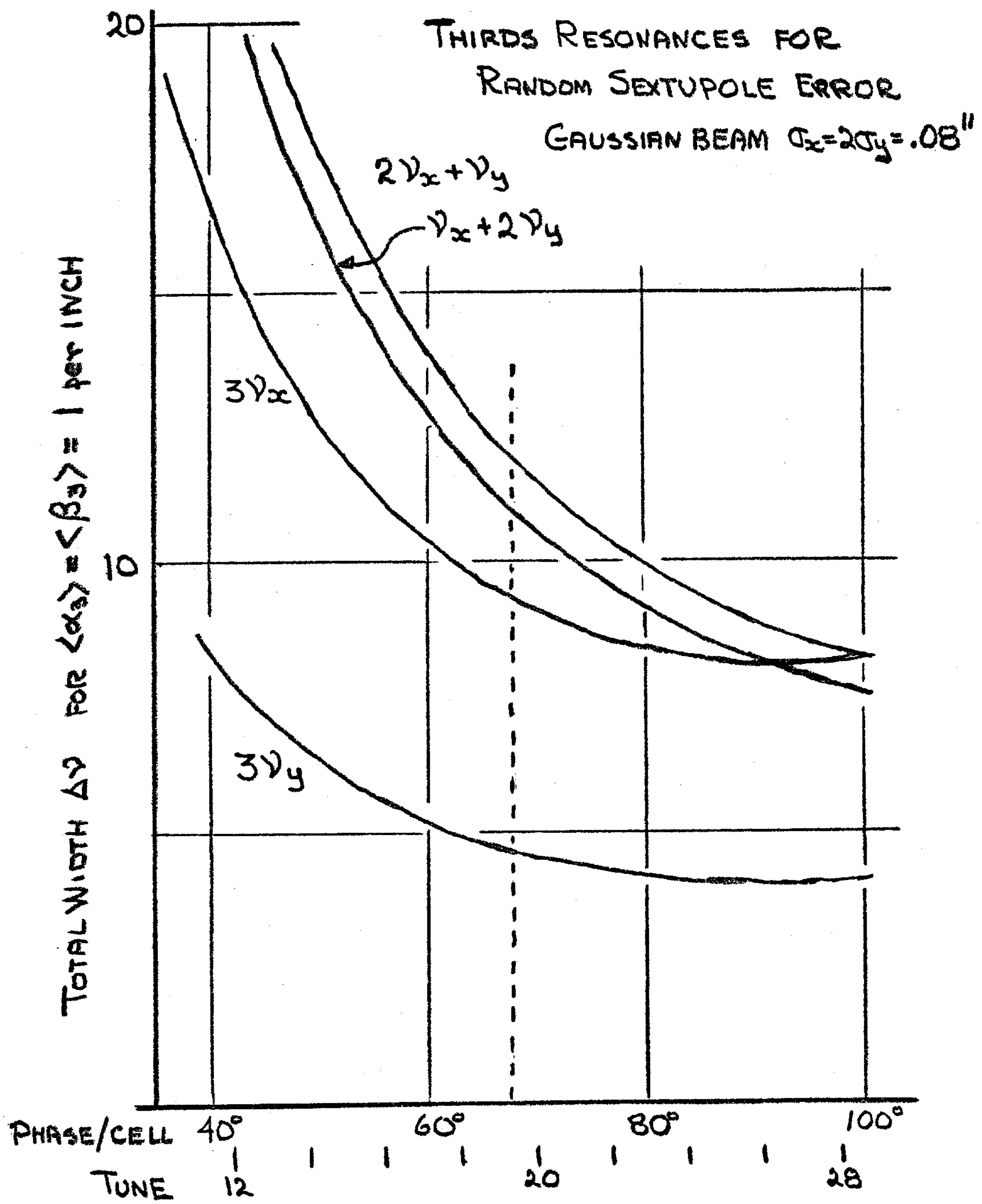


Fig. 7. Widths of third integral resonances at 200 GeV, as a function of phase advance in the normal cell.

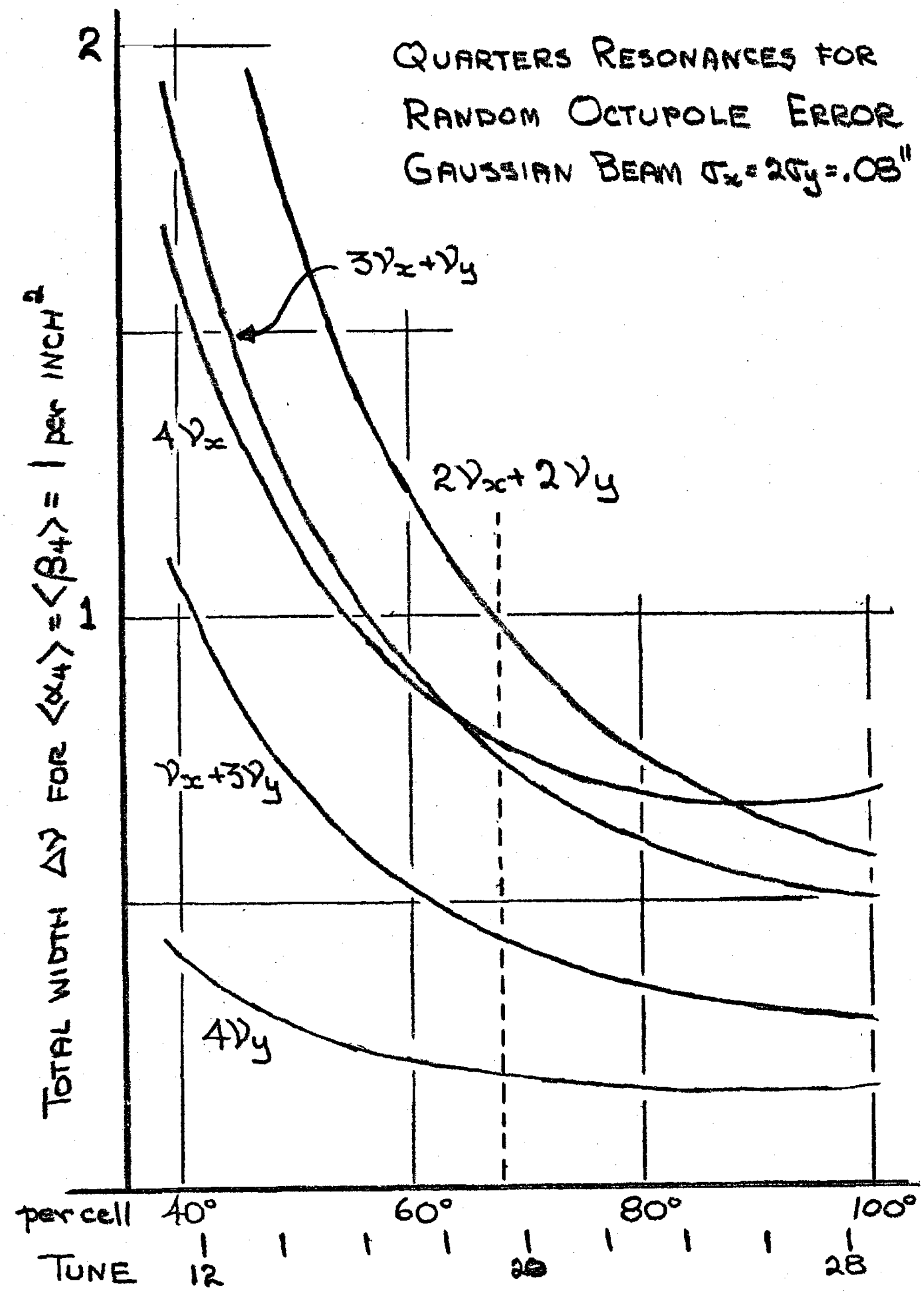


Fig. 8. Widths of fourth order resonances at 200 GeV, as a function of phase advance per cell.

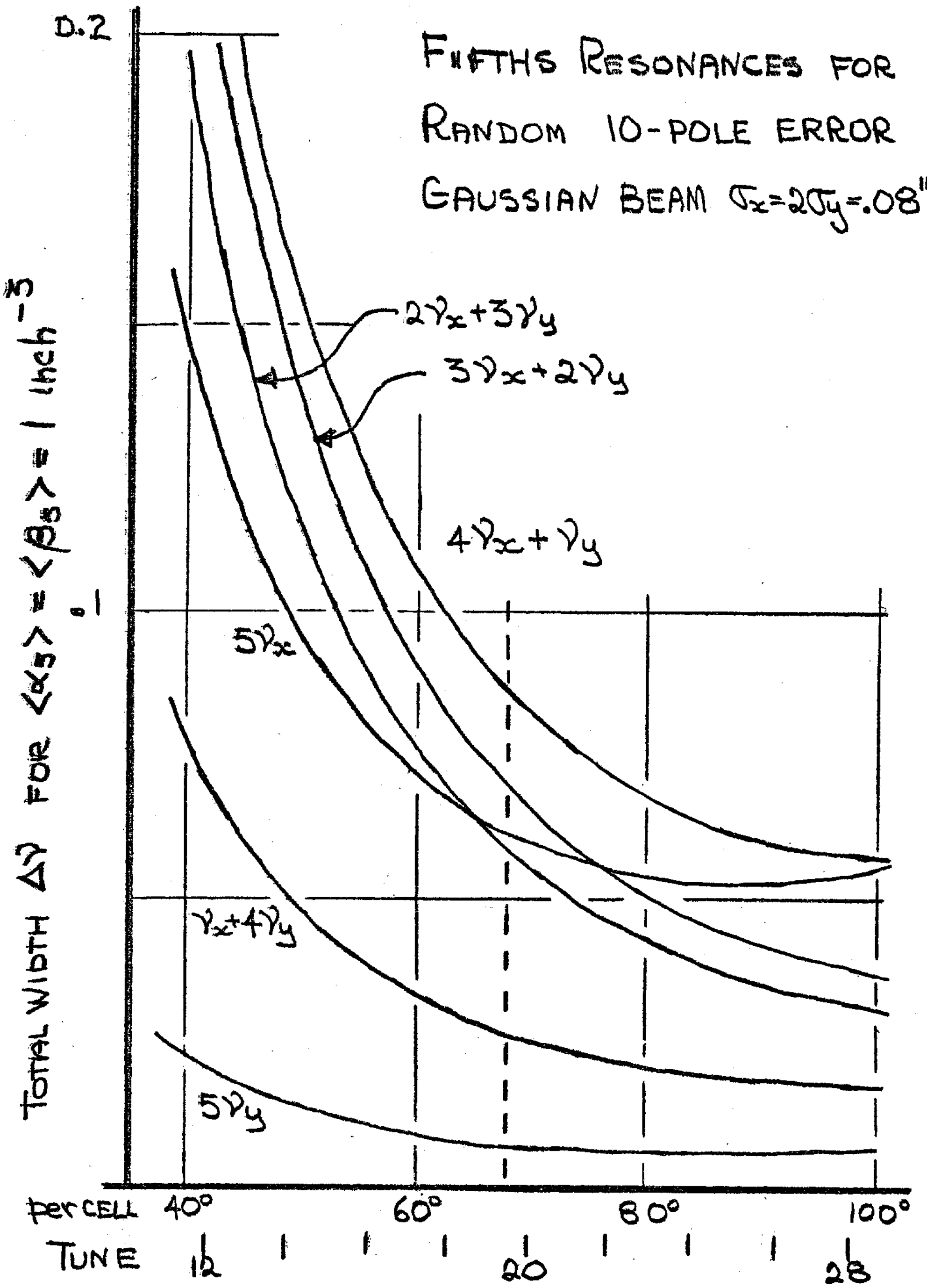
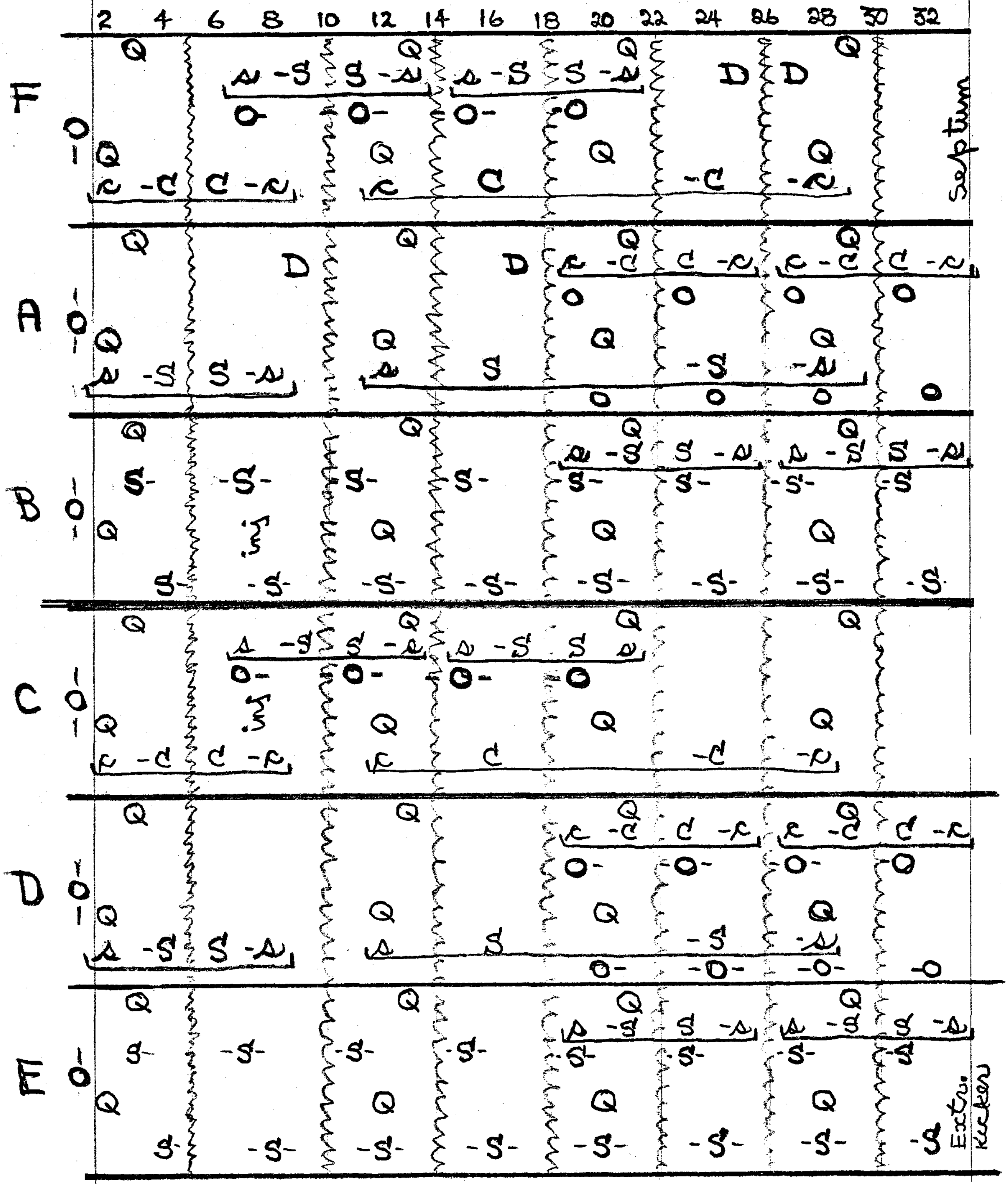


Fig. 9. Widths of fifth order resonances at 200 GeV, as a function of phase advance per cell.

Fig 10. LOCATION OF ADJUS./CORR. ELEMENTS



D - dipole
Q - quad
O - octupole
S sextupole
Ѕ .6 strength sex.
C crossed sex.
ϲ .6 strength.

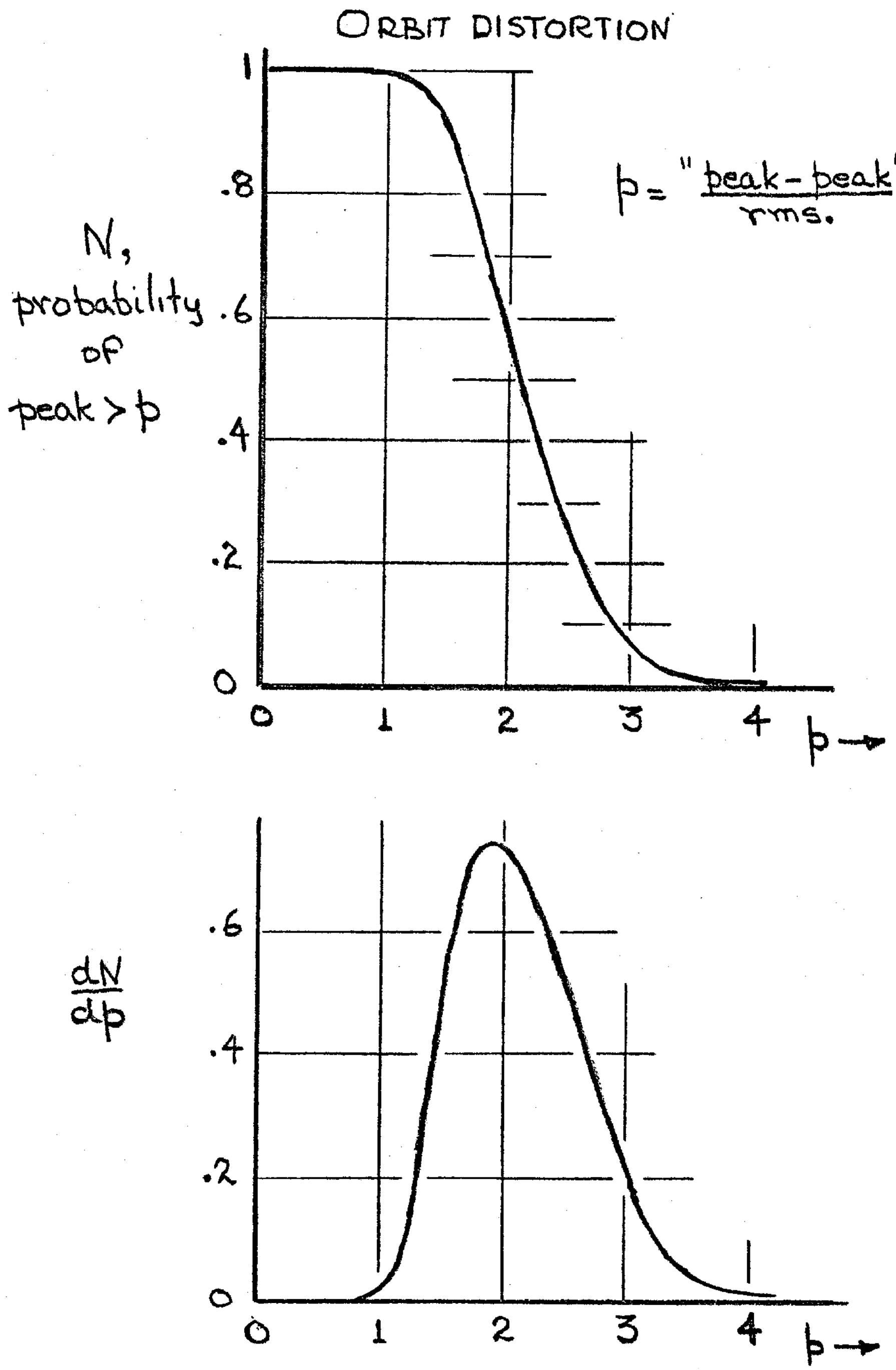


Fig. 11. The upper graph shows the probability that the peak-to-peak orbit distortion will exceed the rms orbit distortion by a factor p. The differential distribution is given in the lower graph.